Power Allocation for Coordinated Multi-cell Systems with Imperfect Channel and Battery-Capacity-Limited Receivers

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Abstract—This letter studies the transmit power allocation in downlink coordinated multi-cell systems with the battery-capacity-limited receivers, where the battery level of receivers is considered. The power allocation is formulated as an optimization problem to maximize the minimum signal-to-interference noise ratio of users under the per-base station power constraints and the feasible maximum received data rate constraints determined by the receiver battery level. The optimal solutions are derived by the proposed monotonic optimization technique based algorithm. The proposed algorithm can extend the battery lifetime of the receivers with lower battery level. Simulation results illustrate the performance of the proposed algorithm.

Index Terms—coordinated multi-cell; power allocation; battery capacity limited; battery level constraint

I. INTRODUCTION

With the constantly increasing demand in wireless data rate, the required power at receivers for decoding data is increasing, leading to a battery depletion problem for receiver terminals [1]. To tackle the above problem, the researchers mainly focus on the following two aspects: i) to charge the battery of the receiver terminals by energy harvesting technology, which has been widely studies in many scenarios recently [2-5], and ii) to develop new strategies for allocating resource by considering the battery status of the receiver terminals, and it has been firstly studied in [6] for the multi-user broadcast systems.

This letter focuses on the battery depletion problem for the coordinated multi-cell systems, which is an important network architecture to improve spectral efficiency. On the one hand, we should notice that, considering that each base station (BS) has its own power amplifier, per-BS power constraints is a practical scenario in the coordinated multi-cell systems [7]. In this situation, the application of the scheme in [6] to the coordinated systems may exceed the maximum transmit power of some BSs, which cannot be realized in practice. On the other hand, compared with the traditional resource allocation strategies in the coordinated multi-cell systems, the strategies considering the receiver battery level will be constrained by the feasible maximum received data rate, which is determined by the receiver battery level [6].

Our goal in this letter is to provide a power allocation scheme to avoid the battery depletion of receivers in the coordinated multi-cell systems. With the proposed scheme, the transmit power of BSs will be controlled and the data rate at receivers, especially the receivers that with lower battery level, will be restricted, preventing the requisite battery consumption to exceed the battery level of receivers. Different from [6], where each battery-capacity-limited user is served by one BS, the goal of our work is to investigate the power allocation scheme for the battery-capacity-limited user which is coordinately served by multiple BSs. As compared to the scheme in [6], the per-BS power constraints but not the total BS power constraint should be satisfied, which makes the power allocation more challenging as they add the transmit power vectors coupling by the maximum transmit power of each BS. We propose a monotonic optimization technique based algorithm [8] to solve the optimization problem. Meanwhile, considering that the channel link is usually rate-limited in practice, the channel state information (CSI) and the receiver battery status information (BSI) required at the BSs are assumed to be imperfect. With the proposed power allocation scheme in this letter, the battery lifetime of the receivers can be extended when the receiver battery level is not sufficient.

II. SYSTEM MODEL

Consider a downlink coordinated multi-cell system, where $N$ BSs are coordinated to simultaneously serve $K$ single-antenna battery-capacity-limited users. A central unit is deployed to implement the coordination among the BSs. It collects the user data and CSI from each BS over the backhaul links, and designs the precoding matrices and the transmit power matrices. Each BS is equipped with $n_k$ antennas, and the total number of the transmit antennas is denoted as $N_t = N n_t$. For the sake of simplicity, we index the BSs and the users by $n \in \mathcal{N} = \{1, \cdots, N\}$ and $k \in \mathcal{K} = \{1, \cdots, K\}$, respectively.

A. Signal Model

Denote $\mathbf{h}_{nk} \in \mathbb{C}^{1 \times n_k}$ ($\forall n \in \mathcal{N}, k \in \mathcal{K}$) as the CSI from BS $n$ to user $k$, and $\mathbf{h}_k = [\mathbf{h}_{1k}, \cdots, \mathbf{h}_{Nk}]$ as the CSI from all BSs to user $k$, whose entries are independent and identically distributed (i.i.d.) complex Gaussian variables with zero mean and unit variance. Assume that the BSs send the modulated data $s_k$ to user $k$ by being preprocessed by the
precoding vector $f_k \in \mathbb{C}^{N_k \times 1}$ ($\forall k \in \mathcal{K}$), and that $s_k$ ($k \in \mathcal{K}$) are statistically independent with zero mean and $E|s_k|^2 = 1$. The received signal at user $k$ is given by

$$y_k = h_k f_k s_k + \sum_{t \in \mathcal{K}, t \neq k} h_t f_t s_t + n_k$$

(1)

where the second term on the right-hand side is the multi-user interference (MUI), and $n_k \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise variable at user $k$.

The precoding vector $f_k$ adopts the zero-forcing precoding, which is derived with the CSI acquired at the BSs. Meanwhile, the CSI acquired at the BSs, denoted as $\hat{h}_k$, are assumed to be imperfect due to the CSI error $\Delta_k$ such that $h_k = \hat{h}_k + \Delta_k, \forall k \in \mathcal{K}$. Therefore, $f_k$ is defined as

$$f_k = \sqrt{p_k} U_k \hat{H}_k^H \hat{h}_k, \forall k \in \mathcal{K}$$

(2)

where $p_k \geq 0, U_k$ denotes the orthogonal basis of the null space of $\hat{H}_k = [\hat{h}_1^T, \cdots, \hat{h}_{k-1}^T, \hat{h}_{k+1}^T, \cdots, \hat{h}_K^T]$, and $p = [p_1, \cdots, p_K]$ represents the transmit power vector.

With $f_k$ defined as (2), (1) is simplified as

$$y_k = h_k f_k s_k + \sum_{t \in \mathcal{K}, t \neq k} \hat{h}_t f_t s_t + \sum_{t \in \mathcal{K}, t \neq k} \Delta_k f_t s_t + n_k$$

$$= \sqrt{p_k} h_k v_k s_k + \sum_{t \in \mathcal{K}, t \neq k} p_k \Delta_k v_t s_t + n_k$$

(3)

It can be seen that the MUI cannot be eliminated completely, due to the CSI acquired at BSs are imperfect. Therefore, by defining $a_k = |\hat{h}_k|^2, b_{kt} = |\Delta_k v_t|^2, \forall k, t \in \mathcal{K}, t \neq k$, the signal-to-interference noise ratio (SINR) at user $k$ is

$$\rho_k(p) = \frac{a_k p_k}{\sigma^2 + \sum_{t \in \mathcal{K}, t \neq k} b_{kt} p_t}$$

(4)

Then, the data rate received at user $k$ is $r_k = \log_2 (1 + \rho_k(p))$.

**B. Battery Model**

The battery level of users are considered for allocating the transmit power, and the BSI acquired at the BSs are imperfect due to the quantization error or the feedback delay error, denoted as $C_k, \forall k \in \mathcal{K}$. It constraints the energy consumption for receiving the signals, including the front-end consumption $P_{c,k}$ and the decoding consumption $P_{dec,k}$. Denote $T$ as the duration of each frame and $E_k$ as the available energy to be consumed by the user $k$. Thus, the following constraint should be satisfied,

$$T \cdot (P_{c,k} + P_{dec,k}) \leq E_k$$

(5)

Generally, $E_k$ is set to a fraction of $\hat{C}_k$ (i.e., $E_k = \alpha_k \hat{C}_k (0 \leq \alpha_k \leq 1)$), such that the battery could provide enough energy (for the power consumption of screen and background programs) to guarantee the receiver’s regular work. Furthermore, $P_{c,k}$ depends on the channel quality, and $P_{dec,k}$ depends on the data rate $r_k$ and the bit error rate (BER). In this letter, we assume $P_{c,k}$ is a constant as the CSI is available, and consider the effect of $r_k$ on $P_{dec,k}$ with the fixed BER, denoted as $P_{dec,k} = f(r_k)$. There is not any model of $f(r_k)$ extensively recognized and accepted by the research community. Here, we just present one example proposed in [9], i.e., $f(r_k) = \tau_k e^{2\pi k r_k}$, where the constants $\tau_k (i = 1, 2)$ denote the decoder efficiency. The formula (5) can be rewritten as $T \cdot (P_{c,k} + f(r_k)) \leq E_k$, from which the upper bound of $r_k$ on the maximum data rate to be supported by user $k$ $r_{max,k}$ can be derived, i.e.,

$$r_k \leq r_{max,k} = f^{-1}(E_k), \forall k \in \mathcal{K}$$

(6)

where $f^{-1}(.)$ denotes the inverse function of $f(.)$. Corresponding to the above model [9], it is expressed as $r_{max,k} = \max\{\ln(E_k/T - P_{c,k})/\tau_{k1}/\tau_{k2}, 0\}$. The formula (6) reveals that the receiver battery level constraints the received data rate.

**III. POWER ALLOCATION**

**A. Problem Formulation**

To avoid the battery depletion in the coordinated multi-cell system, the following two types of constraints should be considered for allocating the transmit power. One is the received data rate constraint, which is related to the receiver battery level and is shown in (6). Another is the per-BS power constraints, which should be always satisfied for the coordinated multi-cell system and is expressed as

$$\sum_{k \in \mathcal{K}} c_n k p_k \leq P_{tx,n}, \forall n \in \mathcal{N}$$

(7)

where $c_n k = v_k^H B_n v_k$, $B_n = diag(0_{(n-1)n}, I_n, 0_{(N-n)n})$, and $P_{tx,n}$ denotes the maximum transmit power of the BS $n$. Specifically, the power allocation scheme so as to provide the fairness performance among users under the above constraints, with imperfect CSI and BSI, can be formulated as the following max-min SINR optimization problem,

$$(P1) : \max_{p} \min_{k \in \mathcal{K}} \rho_k(p)$$

s.t. $\sum_{k \in \mathcal{K}} c_n k p_k \leq P_{tx,n}, \forall n \in \mathcal{N}$

(8)

$$\log_2 (1 + \rho_k(p)) \leq r_{max,k}, \forall k \in \mathcal{K}$$

$$\log_2 (1 + \rho_k(p)) \geq r_{min,k}, \forall k \in \mathcal{K}$$

$$p_k \geq 0, \forall k \in \mathcal{K}$$

where (c) denotes the downlink rate constraint, and $r_{min,k}$ denotes the minimum rate requirement of user $k$.

**B. Problem Solution**

**Remark(Feasibility):** Note that if $r_{min,k}$’s are too large, there may not exist a feasible solution to problem (P1). To guarantee the feasibility of problem (P1), the following two conditions should be satisfied: i) $r_{min,k} \leq r_{max,k}$, and ii) $p_k$ that makes each user rate achieves $r_{min,k}$, exists and satisfies (7). The procedure of the feasibility check is shown as follows.

**Procedure 1 Feasibility check of $r_{min,k}$ in problem (P1)**

1. Check whether $r_{min,k} \leq r_{max,k}$ or the maximum eigenvalue of matrix $G$ is smaller than 1, where the elements of $G$ are

$$G_{kt} = \begin{cases} 0, & k = t \\ (2^{(r_{min,k} - 1)} - 1) b_{kt}/a_k, & k \neq t \end{cases}$$

If one of them is unsatisfied, $r_{min,k}$’s are infeasible. Otherwise, go to step 2.

2. Compute $p_k$ according to $p = (I - G)^{-1} l$, where $l$ is a $K \times 1$ vector with elements $l_k = (2^{(r_{min,k} - 1)} - 1)a_k$, and check whether it satisfies (7). If it is satisfied, $r_{min,k}$’s are feasible. Otherwise, $r_{min,k}$’s are infeasible.
To solve the optimization problem (8), we firstly transform the optimal problem (P1) to an equivalent monotonically increasing problem, which is defined as (P2) in the following; then solve problem (P2) by the proposed monotonic optimization technique based algorithm.

Problem (P1) can be rewritten as follows,

\[
(P1'): \max \min_{\mathbf{p} \in \mathcal{P}} \rho_k(\mathbf{p}) \quad \text{s.t.} \quad \mathbf{p} \in \mathcal{P}
\]

with the feasible set \( \mathcal{P} = \{ \mathbf{p} | \sum_{k \in \mathcal{K}} c_{nk} p_k \leq P_{tx,n}, 2^{r_{\text{min},k}} - 1 \leq \rho_k(\mathbf{p}) \leq 2^{r_{\text{max},k}} - 1, \mathbf{p} \geq 0, \forall k \in \mathcal{K}, n \in \mathcal{N} \} \), which is a nonempty polytope on \( \mathcal{R} \).

Inspired by \( \max_{q \in \mathbb{Q}} q = \max_{u \in \mathcal{U}} u \) for \( \forall \mathbb{Q} \subset (0, +\infty) \) with \( \mathcal{U} = \{ u | 0 \leq u \leq q, q \in \mathcal{Q} \} \), we further transform the problem (P1’) equivalently to

\[
(P2): \max_{\mathbf{u} = [u_1, \ldots, u_K]} \Phi(\mathbf{u}) \triangleq \min_{\mathbf{u} \in \mathcal{U}} u_k 
\]

where the feasible set \( \mathcal{U} = \{ u | 0 \leq u_k \leq \rho_k(\mathbf{p}), \forall k \in \mathcal{K}, \mathbf{p} \in \mathcal{P} \} \). It is noticed that \( \Phi(\mathbf{u}) \) is monotonically increasing in normal set \( \mathcal{U} \), i.e., for \( \forall u \geq u, u \in \mathcal{U} \), \( \Phi(u) \geq \Phi(u) \). Thus, the optimal solution of (P2), denoted as \( \mathbf{u}^* = [u_1^*, \ldots, u_K^*] \), must be achieved at \( \mathbf{u}_k = \rho_k(\mathbf{p}), \forall k \in \mathcal{K} \). Denote \( \mathbf{p}^* \) as a power vector corresponding to the optimal solution \( \mathbf{u}^* \), i.e., \( u_k^* = \rho_k(\mathbf{p}^*), \forall k \in \mathcal{K} \). It is clearly that \( \mathbf{p}^* \) is the optimal solution of problem (P1) and can be found by solving the \( \mathcal{K} \) linear equations \( u_k^*(\sigma^2 + \beta \sum_{j \in \mathcal{K}, j \neq k} b_{kj} p_j - a_k p_k) = 0 \)

with \( \mathcal{K} \) variables \( p_1, \ldots, p_K \). The \( \mathcal{K} \) equations are linearly independent with probability 1 as the independence of the coefficients of \( p_k(k \in \mathcal{K}) \), revealing that there is a unique solution. Hence, the problem (P1) and (P2) are equivalent with each other. In the following, we focus on how to solve the problem (P2).

Considering the monotonically increasing of \( \Phi(\mathbf{u}) \), we propose a monotonic optimization technique [8] based algorithm to derive the optimal solution of problem (P2). The basic idea of the proposed algorithm is to construct the outer polyblock \(^1\) of \( \mathcal{U} \) iteratively (denoted as \( \mathcal{R}_i \) in the \( i \)th iterative) approximating \( \mathcal{U} \), i.e., \( \mathcal{R}_0 \supseteq \mathcal{R}_1 \supseteq \cdots \supseteq \mathcal{U} \), until the optimal vertex in the outer polyblock of \( \mathcal{U} \) lies in \( \mathcal{U} \). Specifically, firstly, an initial outer polyblock \( \mathcal{R}_0 \) of \( \mathcal{U} \) is constructed with one vertex \( \mathbf{u}_0 = [u_0, \ldots, u_K] \) (the value is initialized) and vertex set \( \mathcal{T}_0 = \{ \mathbf{u}_0 \} \). Then, verify that whether \( \mathbf{u}_0 \) is the optimal solution. If \( \mathbf{u}_0 \notin \mathcal{U} \), problem (P2) is solved and \( \mathbf{u}^* = \mathbf{u}_0 \); otherwise, construct a smaller outer polyblock \( \mathcal{R}_1 \subset \mathcal{R}_0 \) of \( \mathcal{U} \) with vertex set \( \mathcal{T}_1 = \{ v_1, \ldots, v_1K \} \), where each vertex \( v_{1j} = u_{0j} - (1 - \lambda_0) u_{0j} e_j, \forall j \in \mathcal{K} \) (the calculation of \( \lambda_0 \) will be introduced below), and then find the vertex \( \mathbf{u}_1 \) in \( \mathcal{T}_1 \) that maximizes the objective function of problem (P2), i.e., \( \mathbf{u}_1 = \arg \max \{ \Phi(\mathbf{u}) | \mathbf{u} \in \mathcal{T}_1 \} \). The above procedure is repeated until an optimal solution is found, or in other words, the algorithm will terminate at the \( i \)th iteration if \( \mathbf{u}_i \in \mathcal{U} \).

\(^1\)Given any finite set \( \mathcal{T} = \{ v_i | i = 1, \cdots, K \} \), the union of all the boxes \( [0, v_i] \), denoted as \( \mathcal{R} \), is a polyblock with vertex set \( \mathcal{T} \). \( \mathcal{R} \) is an outer polyblock of \( \mathcal{U} \) if \( \mathcal{R} \supseteq \mathcal{U} \) [8].
It is noted that Algorithm 2 is an iteration algorithm and the convergence of it for any $\delta > 0$ can be proved, similarly as the Proposition 3.9 in [8], by revealing the Lipschitz continuity of $\Phi(u)$. As the optimal solutions are derived with several iterations, Algorithm 2 has relatively high complexity for applying in practical system. However, it helps us verify the feasibility and the performance of the proposed power allocation scheme, and provides the performance baseline for low-complexity algorithms that we will study in the future.

IV. SIMULATION RESULTS

This section presents some simulation results to evaluate the performance of the proposed power allocation scheme. In the following, the scheme with imperfect BSI and perfect BSI is denoted as "case1" and "case2", respectively. We compare them with the power allocation with non-battery level constraint, denoted as "case3".

In the simulation, we consider a coordinated multi-cell system where three BSs with four antennas each are coordinated to simultaneously serve three users, i.e., $N = 3$, $K = 3$, $n_t = 4$. The imperfect of CSI is assumed to be result from the channel quantization. We use the random vector quantization codebook, and apply the per-cell codebook based independent codeword selection scheme [10] (with perfect phase information), which is proposed for the coordinated multi-cell system, to quantize the CSI. The size of each codebook is $2^{n_t}$, and the dimension of each codeword in each codebook is $1 \times n_t$. Each BS is assumed to have the same maximum transmit power constraint, and the minimum rate requirement of each user is 1bps/Hz. The signal to noise ratio is set to 15dB.

Assume that the battery capacity of each receiver is $C_{\text{max},k} = 3000 \ (k \in \mathcal{K})$ Joule, and the duration of each frame is $T = 1 \text{ms}$. We assume that the proposed algorithm is trigged when the battery level is lower than 20% of the battery capacity. The BSI is quantized using the uniform quantization method within $[0,0.2C_{\text{max},k}]$, which is uniformly divided into 16 subrange. The front-end consumption is $P_{e,ck} = 0.2W$ and the decoding consumption is computed by $P_{\text{dec},ck} = \tau_{1k} e^{2\tau_{2k}}$, where $\tau_{1k} = 1000$ and $\tau_{2k} = 10/11$. The available energy to be consumed by the user $E_k$ is set to 0.1 of the battery level.

Fig. 1 illustrates the variation of average residual battery level (averaged over the three users) and user rate for each user over 70 frames. The initial battery level of each receiver is assumed to be 20% of the battery capacity. The BSI is marginally decreased data rate.

![Fig. 1. The variation of average residual battery level (averaged over the three users) and user rate](image-url)

REFERENCES


