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Research article

A comparative analysis of stochastic models for stock price forecasting: The influence of historical data duration and volatility regimes

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Abstract: Accurate stock price forecasting is essential for informed financial decision-making. This study presents a comparative analysis of four foundational stochastic models—Geometric Brownian Motion (GBM), the Heston Stochastic Volatility model, the Merton Jump-Diffusion (MJD) model, and the Stochastic Volatility with Jumps (SVJ) model—each formulated to capture distinct features of financial market dynamics. Utilizing maximum likelihood estimation (MLE) for parameter calibration and Monte Carlo simulation for forecasting, we assessed model performance over varying historical calibration windows (3-month, 6-month, and 1-year) and a 3-months prediction horizon. Empirical findings demonstrate that the SVJ model consistently achieves superior predictive performance, as quantified by root mean square error (RMSE) and mean absolute percentage error (MAPE), across assets with both low and high volatility profiles. Moreover, the analysis reveals that for low-volatility stocks, such as AAPL and MSFT, a 1-year calibration window yields lower forecast errors, whereas for high-volatility stocks, such as TSLA and MRNA, a 6-month calibration window provides improved forecasting accuracy. These results highlight the importance of selecting model structures and estimation periods that align with the underlying volatility characteristics of the asset.

Keywords: Geometric Brownian Motion; Heston Stochastic Volatility; Merton Jump-Diffusion; Stochastic Volatility with Jumps; stock price forecasting; Monte Carlo Simulation

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1. Introduction

The field of stock market forecasting is fundamental to financial mathematics and quantitative analysis. Developing precise predictive models is crucial for a wide range of financial applications and strategic planning.Traditionally, Geometric Brownian Motion (GBM) has been widely employed for modeling stock prices due to its simplicity and the assumption that prices follow a random walk with constant volatility. However, GBM's limitations, particularly its inability to account for the stochastic nature of volatility observed in real markets, have spurred the development of more sophisticated approaches.

The Heston model, introduced by Steven Heston in 1993, incorporates stochastic volatility into stock price modeling, addressing key shortcomings of GBM. Similarly, the Merton Jump Diffusion model (MDJ), proposed by Robert Merton in 1976, extends the Black-Scholes framework by accommodating sudden, large changes in stock prices, known as jumps. These advanced models are particularly valuable in representing the complex dynamics of financial markets, where stock prices do not follow simple continuous paths but instead exhibit features such as volatility clustering and abrupt price changes due to external shocks (Campbell et al. (1998); Carr and Wu (2004); Carr et al. (2003)).

The quest to decipher and forecast stock market movements has long been a focal point of financial research, drawing scholars and practitioners from diverse fields such as mathematics, economics, and computer science. This enduring interest is fueled by the potential for substantial financial gains and the intellectual challenge posed by the intricate dynamics of financial markets (Black and Scholes (1973)). As the financial landscape continues to evolve, so do the analytical tools and methodologies employed to scrutinize, and predict market behavior (Duffie et al. (2000); Escobar and Gschnaidtner (2016); Constantinides and Richard (1978)).

At the heart of stock market prediction lies the concept of asset pricing, a fundamental challenge in financial mathematics (Fama (1970)). Over time, researchers have developed a variety of models to address this challenge, with four approaches gaining particular prominence: the GBM model, the Heston model, the MJD model, and the Stochastic Volatility with Jumps (SVJ) model. These frameworks, along with their counterparts, utilize stochastic differential equations (SDEs) to represent the inherent volatility and uncertainty of stock prices (Hull and White (1987); Heston (1993); Merton (1976)).

The GBM model, which serves as the foundation for the influential Black-Scholes-Merton (BSM) framework, has been a cornerstone of financial modeling since its inception (Black and Scholes (1973); Reddy and Clinton (2016); Suganthi and Jayalalitha (2019); Stojkoski et al. (2020)). This model postulates that stock prices follow a random walk characterized by constant drift and volatility. Its comparative simplicity and analytical solvability have made it a widely adopted tool for option pricing and basic stock price forecasting. However, the GBM model is not without limitations. Its assumption of constant volatility fails to align with observed market behavior, where volatility tends to cluster and exhibit mean-reversion (Cont (2001)). Moreover, it does not account for sudden jumps in stock prices that can occur due to unexpected events (Merton (1976); Pelet (2003); Ohnishi (2003); Kou (2002)).

To address these shortcomings, more sophisticated models have been developed. The Heston model builds upon the GBM framework by introducing stochastic volatility (Heston (1993)). This model employs two coupled SDEs to represent both stock price and volatility dynamics. Its ability to reflect volatility clustering and mean-reversion has made it a popular choice for option pricing and market prediction. The MJD model tackles another limitation of the GBM approach by incorporating discrete jumps in stock prices (Merton (1976); Matsuda (2004); Mandelbrot (1997)). This feature makes it particularly adept at modeling the impact of unexpected events on market dynamics. The MJD model has found widespread application in stock market forecasting, especially in predicting the effects of rare but significant events on stock prices. Empirical studies have demonstrated that the inclusion of jumps in the model leads to more accurate forecasts during periods of high volatility or market disruption. For example, during the 2008 financial crisis, models incorporating jump diffusion processes were able to more precisely reflect sharp declines in stock prices than those based solely on GBM or stochastic volatility models without jumps (Gatheral (2011); Pelet (2003); Ohnishi (2003); Antwi et al. (2020); Opondo et al. (2021)).

The SVJ model offers a comprehensive framework by integrating the key features of both the Heston and MJD models (Bakshi et al. (2004); Eraker et al. (2003)). Specifically, it captures stochastic volatility through a mean-reverting variance process and incorporates a Poisson-driven jump component to model sudden, discontinuous price changes. This dual structure allows the SVJ model to effectively account for both volatility clustering and abrupt market movements resulting from unforeseen events. Consequently, it delivers improved accuracy in modeling asset dynamics, particularly under turbulent market conditions such as financial crises or periods of elevated uncertainty. Empirical studies have consistently demonstrated the SVJ model's superior performance in forecasting and option pricing tasks, especially in its ability to capture the nonlinear and heavy-tailed behaviors commonly observed in real-world financial markets (Eraker et al. (2003); Bates (1996); Andersen et al. (2007)).

Comparative analyses of these volatility models have illuminated their relative strengths under various market conditions. For instance, the study of Yun (2011) of the Korean stock market revealed that while the Heston model excelled in normal market conditions, the MJD model proved superior during periods of high volatility, underscoring the importance of jump processes in modeling extreme market movements. Similarly, in Gong and Zhuang (2016) a comprehensive examination of the Chinese stock market demonstrated the Heston model's superiority in option pricing accuracy. However, they also noted the MJD model's effectiveness in representing the leptokurtic features often observed in empirical stock return data.

Recognizing the complementary strengths of these models, researchers have developed hybrid approaches that combine multiple features. The Bates model (Bates (1996)), introduced in 1996 , integrates the stochastic volatility of the Heston model with the jump process of the MJD model. This hybrid approach offers a more flexible and comprehensive representation of stock price dynamics, accounting for both gradual volatility changes and sudden jumps. The Bates model has gained traction due to its ability to convey a wide range of market behaviors. Eraker et al. (2003) found that models incorporating both stochastic volatility and jumps, such as the Bates model, provided a superior fit to S&P 500 index returns compared to models featuring only one of these characteristics. They further noted that allowing for jumps in volatility itself led to even better model performance.

The empirical performance of these models has been subject to extensive scrutiny. A seminal comparative study by Bakshi et al. (2004) examined various stock market forecasting models, including the Heston model. Their research revealed that models incorporating both stochastic volatility and jumps, such as the Heston and Bates models, exhibit superior forecasting accuracy compared to the GBM model, particularly during periods of financial stress or market uncertainty. These findings underscore the critical importance of considering both stochastic volatility and jump components in forecasting models. The development of these hybrid models reflects a broader trend in financial modeling towards increased complexity in an effort to better represent real-world market behavior. However, this increased sophistication comes with its own set of challenges, particularly in terms of model calibration and computational demands. As Gatheral (2011) points out, while more complex models may provide better fits to historical data, they do not necessarily translate to improved out-of-sample predictions. This observation has sparked ongoing debates in the field regarding the trade-off between model complexity and practical utility. Some researchers advocate for simpler models with fewer parameters, arguing that they may be more robust and less prone to overfitting (Zellner (1984)). Others contend that the inherent complexity of financial markets necessitates more sophisticated models to achieve accurate predictions (Cont (2001)).

The financial modeling landscape has undergone a paradigm shift through the strategic integration of machine learning (ML) and deep learning (DL) techniques with traditional stochastic models. Where conventional approaches like the Heston model struggle with computational complexity and rigid parametric assumptions, modern hybrid frameworks leverage neural networks' universal approximation capabilities while preserving financial interpretability. Feng et al. (2018) pioneered this synthesis by employing deep neural networks to dynamically calibrate Heston model parameters, demonstrating superior accuracy over classical optimization methods. This breakthrough inspired subsequent innovations like Kim and Won (2018)'s LSTM-enhanced Bates model, which captures both stochastic volatility and jump dynamics through data-driven memory cells.

The evolution of hybrid modeling has progressed along two key dimensions: architectural sophistication and theoretical consistency (Shynkevich et al. (2017); Singh and Priyanka (2018); Singh and Srivastava (2017); Zhang et al. (2018)) . Architecturally, recent work by Bayer and Stemper (2018) introduces neural differential operators to handle rough volatility processes, while (Gruszka and Szwabiński (2023); Zhong and Enke (2017)) developed attention mechanisms for jump parameter estimation. Theoretically, researchers have addressed the black-box dilemma through financially constrained neural architectures (Mehtab and Sen , 2020) and uncertainty-aware learning paradigms (Sucarrat and Grønneberg, 2020). Practical applications showcase these advances, from Vullam et al. (2023)'s trading strategies achieving statistically significant alpha to Yu et al. (2023)'s cross-domain adaptation of sensor fusion techniques for market regime detection. Current frontiers focus on three transformative directions: (1) physics-informed neural networks that encode no-arbitrage conditions directly into loss functions, (2) transformer-based architectures for high-frequency order book modeling, and (3) federated learning frameworks for privacy-preserving model training across institutions. As comprehensively surveyed by Shah et al. (2022) and Strader et al. (2020), these developments collectively represent a fundamental reimagining of financial econometrics: one where machine learning not only complements but systematically enhances traditional quantitative finance theory.

The COVID-19 pandemic has underscored the importance of incorporating rare, high-impact events into stock market prediction models. This has spurred increased interest in extreme value theory and its applications in finance Embrechts et al. (2013). Models capable of accounting for these "black swan" events are likely to become increasingly crucial in the post-pandemic financial landscape, potentially improving the accuracy of stock market predictions during periods of extreme volatility. Hybrid models which combine multiple features of these traditional models, offer enhanced flexibility in representing complex market dynamics for improved stock price forecasting (Bates (1996); Kim and Won (2018); Hossain et al. (2018)). The integration of machine learning with these financial models is opening new avenues for research and application in stock market prediction (Tsai and Wang (2009); Nassif et al. (2020); Mehtab and Sen (2020); Singh (2022); Andersen et al. (2003)).

Despite these advancements, the fundamental challenge posed by the efficient market hypothesis remains. This theory posits that stock prices reflect all available information, making consistent outperformance through prediction extremely difficult. As such, the quest for accurate stock market prediction continues to be both a practical pursuit and a theoretical conundrum. How does the length of the historical data period used for parameter estimation affect the accuracy of 3-month stock price predictions for the GBM, Heston, and MJD models? Which model provides the most accurate predictions for volatile and less volatile stocks over different time horizons? These inquiries can be addressed through the application of the three stochastic models. The GBM model, characterized by constant volatility and smooth price trajectories, is often criticized for its inability to capture market crashes or volatility clustering, making it a less realistic representation of market dynamics. In contrast, the Heston model introduces stochastic volatility and the leverage effect, enhancing realism but at the cost of greater computational complexity. The MJD model incorporates sudden price jumps, acknowledging the occurrence of discrete price movements; however, its assumption of random jump arrivals may not accurately mirror the dynamics observed in real-world financial markets. A comparative analysis of these models, accompanied by empirical evidence and a discussion of their limitations, would significantly enrich the understanding of their applicability and predictive accuracy in financial forecasting.

This study examines stock price forecasting using four distinct mathematical models: the GBM model, the Heston Stochastic Volatility model, the MJD model and the SVJ model. By comparing the performance of these models across different historical data periods, the study aims to identify the conditions under which each model excels, thereby providing insights into their practical applications in financial forecasting.

2. Stochastic models

Stochastic processes have become an indispensable tool for modelling and analyzing a wide array of real-world phenomena across various domains. In the realm of physics, these processes are employed to characterize the intricate motion of particles, providing valuable insights into the underlying dynamics. Similarly, in the medical field, stochastic processes have proven instrumental in understanding the complex spread of infectious diseases and unraveling the intricacies of genetics. Moreover, these processes have found extensive application in the financial sector, particularly in the context of asset price modelling, where they enable the representation of inherent uncertainties and fluctuations (Øksendal (2003); Quayesam et al. (2024); Kou (2007)).

2.1. Geometric Brownian Motion (GBM)

The GBM, a type of SDE commonly used to model stock prices, is a key component of the Black-Scholes option pricing formula. The GBM is defined in its one-dimensional form as:

$$dY_t = \mu Y_t dt + \sigma Y_t dW_t \tag{1}$$

where μ and σ are the drift and diffusion constants, respectively. The drift term $\mu Y_t dt$ represents the expected rate of return of the stock, while the diffusion term $\sigma Y_t dW_t$ illustrates the random fluctuations in the stock price. Although the GBM is a continuous stochastic process, it can be used to model discretely observed data, such as stock prices, by simulating sample paths using the Euler Maruyama (EM) method.

The solution to the stochastic differential equation (SDE) presented in Equation (1) can be derived by employing Itô's formula, yielding the following expression:

$$Y_t = Y_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right) \tag{2}$$

This is the solution to the SDE for the GMB model (Hull and White (1987)).

2.2. The Heston model

The Heston model (Heston (1993)) introduced by Steven Heston in 1993, is an extension of the traditional GMB framework used to model stock price dynamics. The key innovation of the Heston model lies in its introduction of a stochastic process for the variance, denoted as v_t, which allows volatility to vary dynamically over time. This approach provides a more realistic depiction of stock price behavior compared to models that assume constant volatility, such as the GBM model.

The model is governed by two stochastic differential equations (SDEs) that describe both the stock price and its variance over time. The dynamics of the stock price Y_t are given by the equation:

$$dY_t = \mu Y_t\, dt + \sqrt{v_t} Y_t\, dW_t^Y \tag{3}$$

where Y_t represents the stock price at time t, μ is the drift or expected return of the stock, v_t is the instantaneous variance (volatility squared), and W_t^Y is a Wiener process that introduces randomness into the stock price.

The variance process, which governs the evolution of volatility, follows a Cox-Ingersoll-Ross (CIR) process (Cox et al. (1985)), defined by :

$$dv_t = \kappa(\theta - v_t)\, dt + \xi \sqrt{v_t}\, dW_t^v \tag{4}$$

$$dW_t^v dW_t^Y = \rho dt \tag{5}$$

where several important parameters influence the behavior of volatility. The parameter κ represents the rate of mean reversion, which controls how quickly the variance returns to its long-term average θ. When the variance v_t deviates from θ, the mean reversion mechanism pulls it back towards equilibrium level. If the current variance is higher than the long-term mean, it will tend to decrease, and if it is lower, it will tend to increase, ensuring that the volatility stabilizes over time. The volatility of variance ξ controls the magnitude of fluctuations in volatility itself. A higher ξ implies more erratic volatility paths, while $\xi = 0$ collapses the model to deterministic volatility. Parameter ρ represents the correlation between two Wiener processes.

The long-term mean of the variance, θ, reflects the expected average level of volatility in the market. Over time, the variance fluctuates around this value, allowing the model to represent the tendency of volatility to revert to a certain level. However, this mean reversion is not instantaneous, and the speed of this adjustment is governed by the κ parameter.

2.3. Merton's Jump Diffusion model

Merton's Jump Diffusion Model (MJD) is an extension of the GBM that includes jumps, aiming to better reflect the real dynamics of stock prices, which can exhibit sudden, discontinuous changes. In Merton's model, the stock price Y_t evolves according to the following stochastic differential equation:

$$\frac{dY_t}{Y_t} = \mu dt + \sigma dW_t + \delta(dq_t - \lambda dt), \tag{6}$$

Here, q_t is a compound Poisson process, where the jumps have a log-normal distribution. Specifically, the jumps Z_i are distributed as $\ln(1 + Z_i) \sim \mathcal{N}(\nu, \omega^2)$, where ν and ω are the mean and variance of the log-jumps, respectively (Merton (1976)). The term $\delta(dq_t - \lambda dt)$ in equation (6) represents the adjustment for the expected jump size, ensuring that the model remains a martingale. This adjustment makes the model more realistic by incorporating both continuous fluctuations and sudden, unpredictable jumps. The solution to Merton's model is given by:

$$Y_t = Y_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2 - \lambda\nu\right)t + \sigma W_t + \sum_{i=1}^{J_t} Z_i\right) \tag{7}$$

This formula reflects the combined effects of continuous Brownian motion and discrete jumps on the asset price, making it a versatile tool in financial modeling.

2.4. Stochastic Volatility with Jumps (SVJ) model

The SVJ model, also known as the Bates model, extends the Heston stochastic volatility framework by incorporating discontinuous jumps in asset prices. This combination allows the model to more accurately capture stylized facts of financial markets, such as volatility clustering and sudden, large price movements (Bates (1996); Gruszka and Szwabiński (2023)). The SVJ model describes the joint dynamics of the stock price Y_t and its variance process v_t through the following system of stochastic differential equations:

$$\frac{dY_t}{Y_t} = \mu\, dt + \sqrt{v_t}\, dW_t^Y + (e^Z - 1)\, dN_t, \tag{8}$$

$$dv_t = \kappa(\theta - v_t)\, dt + \xi\sqrt{v_t}\, dW_t^v, \tag{9}$$

In this specification, W_t^Y and W_t^v are correlated Brownian motions with instantaneous correlation ρ, so that $\mathbb{E}[dW_t^Y\, dW_t^v] = \rho\, dt$. The variance process v_t follows a mean-reverting square-root process, as in the Heston model, where κ is the speed of mean reversion, θ is the long-run variance level, and ξ governs the volatility of volatility. The jump component is introduced via a Poisson process N_t with intensity λ, and jump sizes Z are independently drawn from a normal distribution, $Z \sim \mathcal{N}(\mu_J, \sigma_J^2)$. The jump term $(e^Z - 1)\, dN_t$ captures discrete shifts in the asset price, while the stochastic volatility component $\sqrt{v_t}\, dW_t^Y$ captures the persistent and autocorrelated nature of volatility observed in empirical stock returns. By integrating both effects, the SVJ model provides a flexible and realistic framework for modelling stock price dynamics, particularly useful for pricing derivatives, managing risk, and forecasting volatility in markets characterized by both continuous and abrupt movements.

3. Methodology

This study focuses on forecasting stock prices using four distinct mathematical models: the GBM model, the Heston Stochastic Volatility model, the MJD model, and the SVJ model. These models are implemented and evaluated to explore different aspects of the prediction of stock prices and the performance of the model. The study uses real historical stock data, which is segmented into different periods to assess how varying lengths of historical data influence the accuracy of future price predictions. The aim is to evaluate the predictive accuracy of the GBM, Heston, MJD, and SVJ models over a 3-month forecast horizon. The uniqueness of this task lies in the comparison of models based on different historical data periods (3 months, 6 months, and 1 year), leading up to the forecast period.

The hypothesis is that the length of the historical data used for parameter estimation could significantly affect the model's ability to predict future prices.

3.1. Data collection

The accuracy of model predictions in this study is closely tied to the quality of historical stock price data. Data is sourced from Yahoo Finance using the `yfinance` API, a reliable and widely used tool for obtaining financial information. The stock selection includes both volatile and less volatile companies to test the models across diverse market conditions. Volatile stocks, such as Tesla and Meta, exhibit significant price fluctuations, making them ideal for testing models like the Heston, MJD, and SVJ, which are designed to represent complex price dynamics like jumps and stochastic volatility. Conversely, less volatile stocks, such as those from utilities and consumer goods sectors, provide a stable environment for testing the models in more predictable markets, offering a valuable contrast.

Daily adjusted closing prices are collected for each stock. These prices are used to calculate daily returns, which serve as inputs for estimating parameters for the GBM, Heston, MJD, and SVJ models. The historical data spans 1 year, 6 months, and 3 months, enabling a comparison of how different time frames influence model performance.

In this study, a total of 44 stocks are analyzed, categorized into two groups: less volatile and volatile stocks. The less volatile category includes companies such as Microsoft (MSFT), Apple (AAPL), and Johnson & Johnson (JNJ), which are known for their relatively stable price movements. On the other hand, the volatile category includes companies such as Tesla (TSLA), Meta (META), and Airbnb (ABNB), which are characterized by frequent and large price fluctuations. These stocks are selected to provide a representative sample across different sectors, ensuring a comprehensive evaluation of the models' performance under varying market conditions.

Less volatile stocks:

- VOD.L, MSFT, BP.L, LLOY.L, AAPL, IBM, CMCSA, FDX, PEP, MCD, HSBA.L, GS, WMT, LAND.L, GRI.L, SHEL.L, NG.L, TSCO.L, NXT.L, JNJ, PFE, BT.A.L

Volatile stocks:

- TSLA, SHOP, SNOW, ROO.L, PLTR, META, UBER, IDS.L, ABNB, EXPE, BARC.L, ADBE, AMZN, ZM, FOXT.L, SVS.L, ORCL, GOOGL, MKS.L, JD.L, MRNA, BNTX

This diverse stock selection enables the study to assess the performance of the GBM, Heston, MJD, and SVJ models across varying levels of market volatility and uncertainty, providing comprehensive insights into their predictive power.

For the purposes of this study, historical stock price data were collected for the period spanning January 1, 2023, to January 1, 2024. Subsequently, the analysis was extended to encompass a longer time horizon that includes the rare event of the COVID-19 pandemic, to assess the models under extreme market conditions. The dataset was then partitioned into three distinct temporal segments:

- **1-year Period**: January 1, 2023, to January 1, 2024.
- **6-month Period**: July 1, 2023, to January 1, 2024.
- **3-month Period**: October 1, 2023, to January 1, 2024.

This segmentation allows the extraction of model parameters over varying lengths of historical data, providing insight into the optimal data period for accurate predictions. Choosing historical data periods of 3 months, 6 months, and 1 year for the GBM, Heston, MJD, and SVJ models ensures a balanced evaluation of market dynamics. The 3-month period captures short-term volatility and sudden price jumps, making it useful for the MJD and SVJ models. The 6-month period provides a middle ground, allowing the Heston model to capture volatility clustering effectively. The 1-year period helps analyze long-term trends, benefiting the GBM model, which assumes constant volatility and drift. Using multiple time frames improves forecasting robustness by testing models under different market conditions and ensuring practical relevance in stock price prediction.

3.2. Parameter estimation

The first step in implementing the models is the estimation of parameters. For each historical period (1 year, 6 months, and 3 months), key parameters are estimated for each model considered in the study.

For the GBM model, the key parameters are the drift (μ) and volatility (σ). These parameters were estimated using historical log returns of each stock. Table 1 shows the updated average estimated values of μ and σ for less volatile and volatile stocks based on the most recent 1-year, 6-month, and 3-month historical periods.

Table 1. Average GBM parameter estimates for different stock types.

	Less volatile stocks		**Volatile stocks**	
Period	μ	σ	μ	σ
1 year	0.1164	0.1797	0.4734	0.3282
6 months	0.1309	0.1739	0.2888	0.3094
3 months	0.2099	0.1830	0.5489	0.3154

The estimated parameter for volatile stocks exhibits more pronounced variations in both drift and volatility. The drift values increase significantly over the shorter 3-month period (0.5489), indicating strong recent returns for these stocks, while volatility remains consistently high. This behavior is expected for volatile stocks, as shorter time frames often reflect sharper movements in response to market events. In general, these results align with expectations, as volatile stocks tend to show greater sensitivity to recent market conditions, while less volatile stocks maintain steadier performance over time.

For the Heston model, the parameters of interest are the initial volatility (v_0), the speed of mean reversion (κ), the long-term average volatility (θ), the volatility of volatility (ξ), and the correlation between stock price and volatility (ρ). These parameters were estimated using maximum likelihood estimation (MLE) on the historical stock prices. Table 2 presents the average estimated values for each parameter across different stock types and historical periods.

The estimates indicate that volatile stocks have higher values for initial volatility, long-term average volatility, and volatility of volatility, consistent with their greater price variability, which is expected. Volatile stocks typically exhibit larger price fluctuations, so it is unsurprising that the long-term volatility

Table 2. Average Heston parameter estimates for different stock types.

Stock type	Period	v_0	κ	θ	ξ	ρ	μ
Less volatile	1 year	0.0337	1.5068	0.1543	0.2871	-0.0932	0.1164
	6 months	0.0317	1.4450	0.1547	0.2634	−0.0826	0.1309
	3 months	0.0357	1.6336	0.1301	0.2926	−0.0018	0.2099
Volatile	1 year	0.1153	1.6678	0.1703	0.2893	0.1315	0.3282
	6 months	0.1022	1.2320	0.1557	0.2652	0.0415	0.2888
	3 months	0.1070	1.5123	0.1624	0.2914	0.0104	0.5489

(θ) and the volatility of volatility (ξ) are both higher for these stocks. The speed of mean reversion (κ) is generally lower for volatile stocks, especially over the 6-month period, indicating that volatility reverts more slowly to its long-term average, which is also in line with expectations for stocks that experience more dramatic shifts in price behavior.

For the MJD model, the parameters of interest are the jump intensity (λ), the average jump size (ν), and the jump size volatility (ω). These parameters capture the frequency, magnitude, and variability of the jumps in stock prices, which occur in addition to the continuous Brownian motion modeled in traditional stock price models. These parameters were estimated using MLE on historical stock price data. Table 3 presents the average estimated values for each parameter across different stock types and historical periods.

Table 3. Average MJD parameter estimates for different stock types.

	1 year				6 months				3 months			
Stock type	μ	λ	ν	ω	μ	λ	ν	ω	μ	λ	ν	ω
Less volatile	0.1164	0.7114	-0.0081	0.0786	0.1309	0.4673	0.0054	0.0735	0.2099	0.5302	0.0056	0.0681
Volatile	0.3282	0.4912	−0.0006	0.0808	0.2388	0.7176	−0.0009	0.0804	0.5489	0.6734	0.0218	0.0705

The results show some expected differences between less volatile and volatile stocks in terms of jump intensity (λ) and jump size volatility (ω). For less volatile stocks, the jump intensity is highest over the 1-year period (0.7114) and decreases for the 6-month and 3-month periods, suggesting that jump events occur more frequently over longer horizons. The jump size volatility (ω) decreases over shorter periods, with the lowest value at 0.0681 for the 3-month period, which aligns with the expectation that less volatile stocks experience smaller, less frequent jumps.

For the SVJ model, the set of estimated parameters extends those of the Heston model by incorporating jump components to better capture sudden and significant movements in asset prices. The key parameters of interest include the initial variance (v_0), the speed of mean reversion (κ), the long-term mean variance (θ), the volatility of volatility (ξ), and the correlation between asset price and its variance process (ρ). In addition to these, the SVJ model features parameters governing the jump process: the jump intensity (λ), the mean jump size (μ_J), and the standard deviation of jump size (σ_J). These parameters were estimated using MLE based on historical price data across different stock types and time periods. Table 4 reports the average estimated values for each parameter, categorized by volatility regime and historical period.

Table 4. Average SVJ parameter estimates for different stock types and periods.

Stock Type	Period	v_0	κ	θ	ξ	ρ	λ	μ_J	σ_J	μ	σ
Less volatile	1 year	0.0336	1.6549	0.1557	0.2806	0.0257	0.6263	-0.0062	0.2483	0.1055	0.1792
	6 months	0.0319	1.5861	0.1508	0.2948	-0.0065	0.5295	0.0182	0.2345	0.1183	0.1741
	3 months	0.0360	1.3898	0.1715	0.2428	-0.1481	0.5171	-0.0058	0.2405	0.2015	0.1835
Volatile	1 year	0.1153	1.7951	0.1702	0.2746	0.0321	0.5338	-0.0160	0.2565	0.4695	0.3282
	6 yonths	0.1022	1.3532	0.1705	0.2728	0.0008	0.4286	-0.0199	0.3172	0.2856	0.3094
	3 months	0.1070	1.7624	0.1823	0.2928	-0.0278	0.5194	0.0295	0.2136	0.5479	0.3155

The parameter estimates for the SVJ model further reinforce the distinction between less volatile and volatile stocks. As expected, volatile stocks exhibit higher values for initial variance (v_0), long-term average variance (θ), and volatility of volatility (ξ), consistent with their greater susceptibility to rapid and pronounced price changes. Notably, the jump-related parameters also display meaningful differences: volatile stocks generally show slightly higher jump intensities (λ) and larger jump size variability (σ_J), reflecting the more frequent and erratic price discontinuities characteristic of such stocks.

The mean reversion speed (κ) tends to be slightly lower for volatile stocks over certain periods, such as the 6-month interval, implying that deviations from the long-term volatility level persist longer for these stocks. This behavior aligns with the empirical observation that volatility in riskier assets tends to be more persistent. Additionally, the average annualized return (μ) is markedly higher for volatile stocks, especially over shorter horizons, which may reflect the higher risk-return trade-off embedded in these stocks.

3.3. Model implementation

The models are implemented using the estimated parameters from each historical period. This results in three separate sets of predictions for the GBM, Heston, MJD, and SVJ models, each based on parameters from the 3-month, 6-month, and 1-year historical data.

The prediction phase involves running Monte Carlo simulations using each model, generating numerous possible future price paths for the next 3 months (from January 1, 2024, to April 1, 2024). These simulations provide a distribution of potential outcomes, from which summary statistics such as mean predicted price, standard deviation, and confidence intervals are derived. To evaluate the models, the predicted prices are compared to the actual observed prices on April 1, 2024. The root mean square error (RMSE) and mean absolute percentage Error (MAPE) are used as the primary metric for assessing predictive accuracy.

4. Model analysis and findings

In this study, the focus is on how the length of the historical data period affects the performance of the model. By comparing RMSE values across 3-month, 6-month, and 1-year historical periods, this analysis aims to determine the influence of different data lengths on parameter estimation. The results offer insights into how the models respond to varying historical data for both volatile and less volatile stocks.

The models were implemented using parameters estimated from historical data spanning 1 year, 6 months, and 3 months to generate 3-month price predictions. The results, as shown in Figure 1, indicate that the choice of historical data period had a significant impact on the accuracy of predictions, particularly depending on the volatility of the stock being analyzed. Less volatile stocks consistently showed better predictions when a 1-year historical period was used for parameter estimation. In contrast, volatile stocks benefited more from the 6-month historical period, which appeared to strike a balance between reflecting recent market trends and avoiding overfitting to short-term fluctuations.

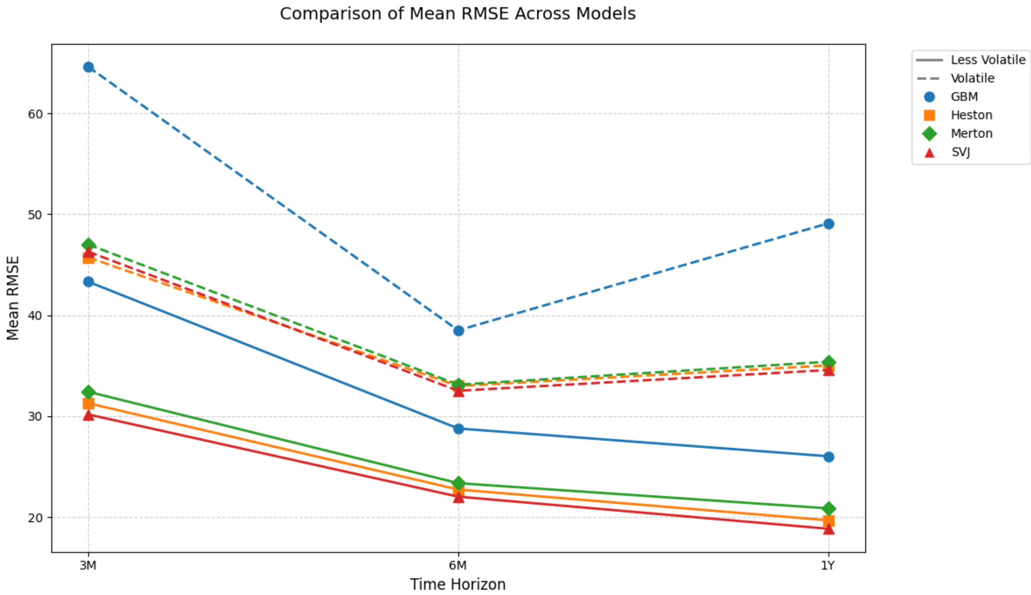


Figure 1. Mean RMSE scores for less volatile (22 stocks) and volatile (22 stocks) companies across 3-month, 6-month, and 1-year estimation periods using the GBM, Heston, MJD, and SVJ models for 3-month predictions. The SVJ model offers better performance compared to other models for both less volatile and volatile companies. The 1-year estimation period yields the best overall performance for less volatile stocks, showing consistently lower RMSE values across all models. For volatile stocks, the 6-month estimation period provides the best overall performance across all models, with lower RMSE values compared to both the 3-month and 1-year periods.

The results for less volatile stocks indicate that the Heston, MJD, and SVJ models consistently outperform the GBM model in terms of RMSE. When evaluating the RMSE for different historical data periods, it becomes evident that the 1-year estimation period provides the best overall performance across all models, particularly for less volatile stocks. The longer historical period allows the models to represent more comprehensive volatility dynamics, resulting in more accurate predictions.

For volatile stocks, the results are more nuanced. Unlike less volatile stocks, where the 1-year estimation period is optimal, volatile stocks demonstrate that the 6-month period yields the best overall performance across all models. The GBM model struggles with higher RMSE values for both the 3-month and 1-year periods, likely due to its inability to adapt to rapid changes in volatility, which are more prevalent in volatile stocks. The Heston and MJD models provide similarly precise estimates, with both offering better performance compared to GBM. The SVJ model is the best performer, consistently providing lower RMSE values than the other models across different historical data periods used for estimation.

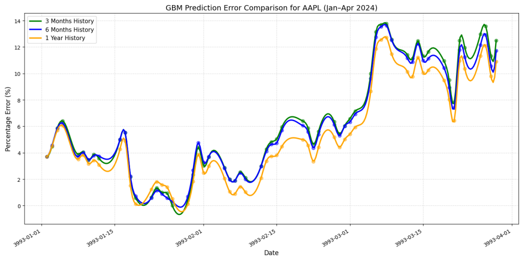
4.1. Case studies on selected stocks

The precision and reliability of the forecast are essential when assessing the performance of financial models. In this study, case studies were carried out on 10 prominent stocks, in a combination of volatile (TSLA, GOOGL, MRNA, META, and AMZN) and less volatile (AAPL, MSFT, IBM, WMT, and FDX) stocks, to evaluate the predictive accuracy of the models. The models were applied to simulate future stock prices using historical data, with predictions compared to the actual stock price movements over a 3- months prediction horizon.

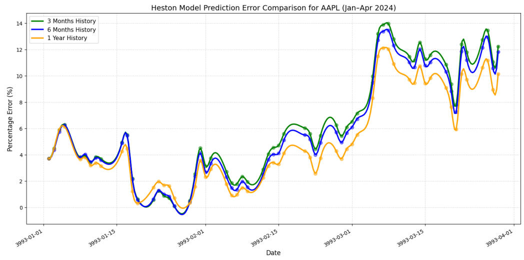
Figures 2 and 3 illustrate the Average Percentage Error (APE) in forecasting stock prices for Apple (AAPL) and Tesla (TSLA), respectively, using four stochastic models: GBM, the Heston model, MJD, and SVJ. The models are calibrated over three different historical data windows—3 months, 6 months, and 1 year. Subfigures (a) to (d) in each figure display the prediction errors for each model under the various calibration windows. For AAPL, a longer historical window—particularly 1 year—consistently results in lower APEs, suggesting improved parameter estimation from longer historical data. In contrast, TSLA shows the best performance with the 6-month window, likely due to its high volatility, where intermediate calibration balances responsiveness and stability.

Subfigures (e), (f), and (g) compare model performance across all historical windows. In both cases, the SVJ model achieves the lowest APEs across all periods, with its advantage being most pronounced for TSLA at the 6-month horizon. This underscores the SVJ model's strength in capturing complex market behaviors such as volatility clustering and sudden price jumps, which are common in tech stocks like AAPL and TSLA. Similar patterns are observed for other high- and low-volatility stocks analysed in this study. To conserve space, we omit the corresponding figures; however, the full set of computed results will be presented in tabular form in a subsequent section. These findings further reinforce the observed trends regarding the influence of historical calibration windows and the relative performance of the SVJ model across varying volatility regimes.

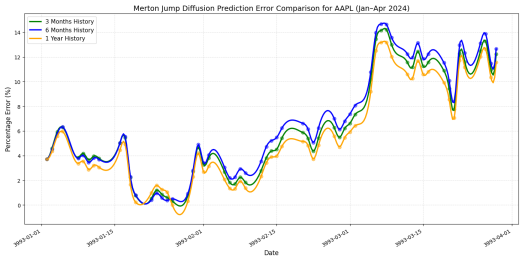
The statistical results comparing the models are presented in table 5-9 for all five selected less volatile stocks (AAPL, MSFT, IBM, WMT, and FDX) using the historical period January 2023 to December 2023. The results illustrated that the SVJ model demonstrates superior performance across all evaluated metrics compared to the other models—GBM, Heston, and MJD. The SVJ model consistently yields the lowest MAPE, RMSE, and absolute price differences across various estimation horizons (3-month, 6-month, and 1-year), highlighting its robustness in capturing both stochastic volatility and sudden jumps in stock prices. In contrast, the GBM model, which assumes constant volatility and lacks jump dynamics, underperforms in nearly all scenarios. The Heston and MJD models, which partially incorporate stochastic behavior or jump components, show moderate improvement over GBM but still fall short of the accuracy provided by the SVJ model. Furthermore, the SVJ model produces narrower confidence intervals, indicating a better fit to actual market behavior.



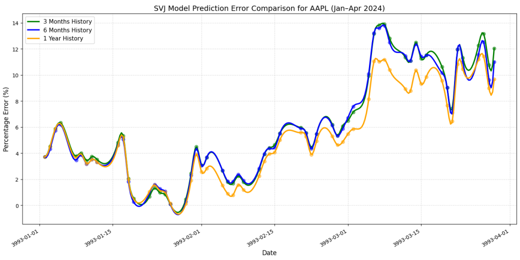
(a) GBM Model



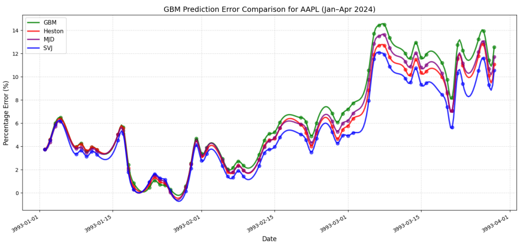
(b) Heston Model



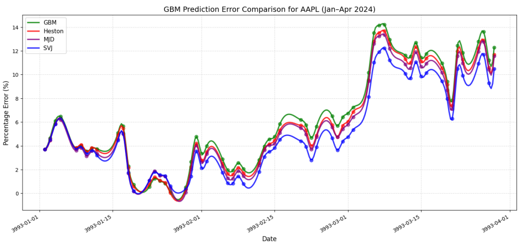
(c) MJD Model



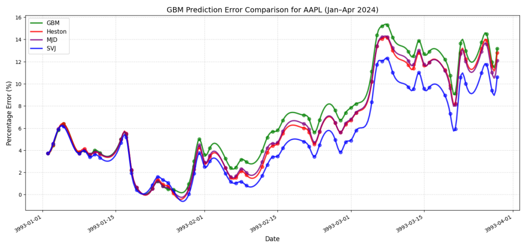
(d) SVJ Model



(e) Average Percentage Error using 3 months historical period



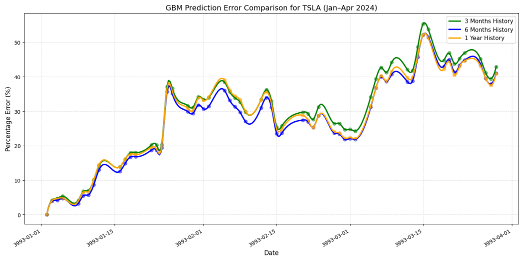
(f) Average Percentage Error using 6 months historical period



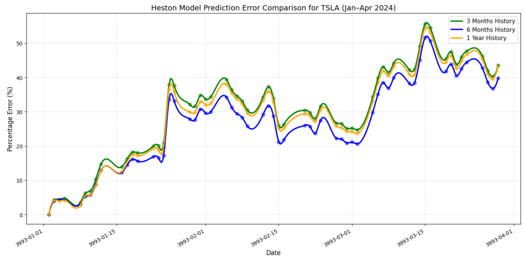
(g) Average Percentage Error using 1 year historical period

Figure 2. Comparison of prediction errors for AAPL stock using four stochastic models—GBM, Heston, MJD, and SVJ—across different historical training periods (3 months, 6 months, and 1 year). Subfigures (a)–(d) display the average percentage error (APE) for each model individually, while subfigures (e)–(g) present model-wise APE comparisons across the specified historical windows. The results highlight the superior performance of the SVJ model and the improved accuracy associated with longer historical periods.

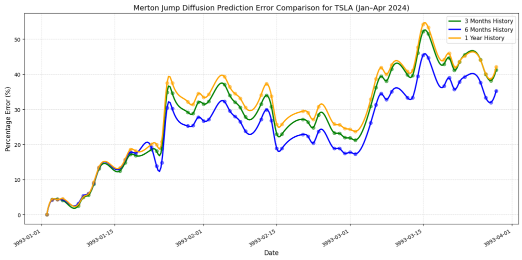
These findings confirm that the inclusion of both stochastic volatility and jump components significantly enhances the predictive accuracy and reliability of stock price modeling.



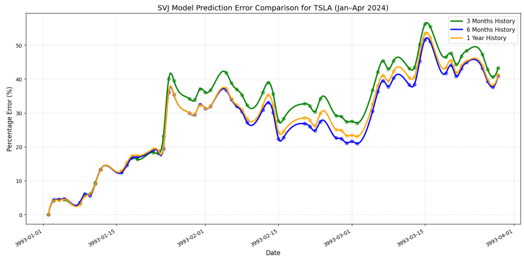
(a) GBM Model



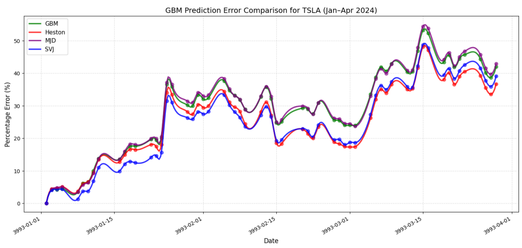
(b) Heston Model



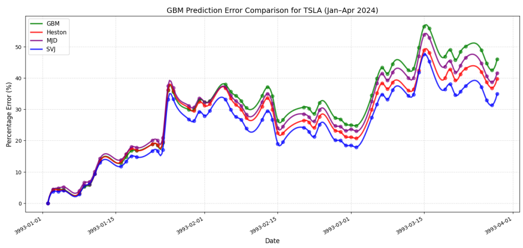
(c) MJD Model



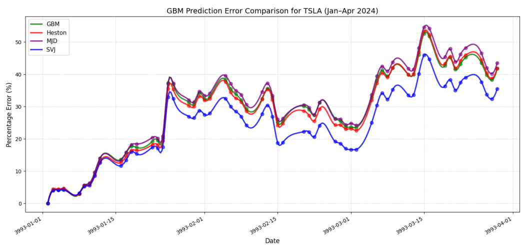
(d) SVJ Model



(e) Average Percentage Error using 3 months historical period



(f) Average Percentage Error using 6 months historical period



(g) Average Percentage Error using 1 year historical period

Figure 3. Comparison of prediction errors for TSLA stock using four stochastic models—GBM, Heston, MJD, and SVJ—across different historical training periods (3 months, 6 months, and 1 year). Subfigures (a)–(d) display the average percentage error (APE) for each model individually, while subfigures (e)–(g) present model-wise APE comparisons across the specified historical windows. The results highlight the superior performance of the SVJ model and the improved accuracy associated with 6 months historical periods.

Table 5. Comparison of models for AAPL using the historical period January 2023 to December 2023.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.1547	8.53%	17.87	194.13	8.34	(177.91, 211.11)	5.34%	13.50
	Heston	0.1547	8.08%	17.20	192.91	8.26	(177.94, 208.57)	4.68%	12.28
	MJD	0.1547	8.35%	17.54	193.74	8.17	(178.34, 209.76)	5.13%	13.10
	SVJ	0.1547	**8.03%**	17.09	193.41	8.71	(176.09, 210.07)	4.95%	12.18
6-M	GBM	0.1889	6.14%	13.27	183.12	9.60	(165.71, 203.46)	−0.63%	2.49
	Heston	0.1889	6.06%	13.03	183.16	9.40	(162.23, 200.36)	−0.61%	2.52
	MJD	0.1889	6.25%	13.46	184.28	9.70	(166.93, 205.26)	−0.01%	3.65
	SVJ	0.1889	**6.02%**	13.02	182.84	9.61	(164.86, 201.78)	−0.79%	2.21
1-Y	GBM	0.1195	5.45%	11.43	116.30	6.65	(103.34, 129.14)	3.11%	7.93
	Heston	0.1195	5.47%	11.45	116.36	6.54	(103.87, 129.22)	3.14%	7.97
	MJD	0.1195	5.36%	11.27	116.00	6.83	(103.85, 129.96)	2.94%	7.61
	SVJ	0.1195	**5.30%**	11.15	115.71	6.65	(103.12, 128.57)	2.79%	7.33

Table 6. Comparison of models for MSFT using the historical period January 2023 to December 2023.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.2036	8.42%	44.64	396.85	22.61	(353.45, 443.60)	8.22%	−3.81
	Heston	0.2036	8.40%	44.28	394.10	22.35	(354.47, 437.18)	7.47%	−6.57
	MJD	0.2036	8.25%	44.31	395.81	22.15	(354.58, 439.71)	7.94%	−4.85
	SVJ	0.2036	**8.17%**	43.18	395.56	23.61	(349.87, 441.80)	7.87%	−5.11
6-M	GBM	0.2123	8.68%	39.33	373.67	22.18	(333.67, 420.98)	1.90%	−26.99
	Heston	0.2123	8.41%	38.19	373.91	21.71	(326.21, 413.95)	1.97%	−26.75
	MJD	0.2123	8.35%	38.04	376.32	22.42	(336.41, 425.12)	2.62%	−24.34
	SVJ	0.2123	**8.27%**	37.21	373.23	22.22	(332.04, 417.49)	1.78%	−27.43
1-Y	GBM	0.2376	8.28%	38.35	367.20	26.38	(316.46, 418.67)	5.14%	−13.42
	Heston	0.2376	8.09%	37.67	367.82	26.01	(319.18, 419.93)	5.30%	−12.81
	MJD	0.2376	8.23%	37.96	366.03	27.09	(318.35, 422.00)	4.82%	−14.60
	SVJ	0.2376	**8.08%**	37.44	365.29	26.29	(316.37, 417.19)	4.62%	−15.34

Table 7. Comparison of models for IBM using the historical period January 2023 to December 2023.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.1519	8.65%	17.84	166.66	7.08	(152.88, 181.11)	8.35%	−8.03
	Heston	0.1519	8.80%	18.14	165.61	7.02	(152.91, 178.87)	7.67%	−9.08
	MJD	0.1519	8.67%	17.89	166.33	6.94	(153.29, 179.95)	8.14%	−8.36
	SVJ	0.1519	**8.57%**	17.66	166.03	7.39	(151.28, 180.11)	7.95%	−8.65
6-M	GBM	0.1437	9.51%	19.36	161.64	6.54	(149.65, 175.38)	5.09%	−13.04
	Heston	0.1437	9.44%	19.17	161.50	6.40	(147.00, 173.03)	5.00%	−13.18
	MJD	0.1437	**9.29%**	18.94	162.44	6.59	(150.47, 176.55)	5.61%	−12.25
	SVJ	0.1437	9.56%	19.44	161.27	6.54	(148.65, 174.04)	4.85%	−13.42
1-Y	GBM	0.1106	7.71%	15.48	109.98	4.96	(100.22, 119.44)	1.51%	−12.30
	Heston	0.1106	7.71%	15.46	109.92	4.86	(100.42, 119.32)	1.47%	−12.36
	MJD	0.1106	7.83%	15.70	109.75	5.10	(100.61, 120.01)	1.35%	−12.54
	SVJ	0.1106	**7.61%**	15.36	109.44	4.98	(99.87, 118.64)	1.16%	−12.84

The statistical evaluation results for the selected volatile stocks—TSLA, GOOGL, MRNA, META, and AMZN—over the historical period from January 2023 to December 2023 are summarized in Tables 10–14.

Table 8. Comparison of models for WMT using the historical period January 2023 to December 2023.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.2189	9.52%	6.42	52.00	3.11	(46.04, 58.41)	−0.39%	−4.29
	Heston	0.2189	9.59%	6.34	51.63	3.08	(46.16, 57.61)	−1.09%	−4.65
	MJD	0.2189	**9.32%**	6.17	51.85	3.06	(46.12, 57.88)	−0.67%	−4.44
	SVJ	0.2189	9.46%	6.25	51.84	3.26	(45.62, 58.29)	−0.69%	−4.45
6-M	GBM	0.1751	8.63%	5.76	52.07	2.53	(47.46, 57.42)	−0.24%	−4.21
	Heston	0.1751	8.48%	5.68	52.07	2.48	(46.51, 56.58)	−0.25%	−4.22
	MJD	0.1751	8.31%	5.57	52.38	2.56	(47.79, 57.89)	0.35%	−3.91
	SVJ	0.1751	**8.26%**	5.48	51.98	2.53	(47.21, 56.94)	−0.42%	-4.30
1-Y	GBM	0.1581	7.74%	5.22	52.78	2.37	(48.10, 57.31)	1.11%	−3.51
	Heston	0.1581	7.67%	5.19	52.75	2.33	(48.20, 57.25)	1.06%	−3.54
	MJD	0.1581	7.90%	5.32	52.66	2.44	(48.28, 57.58)	0.89%	−3.62
	SVJ	0.1581	**7.63%**	5.15	52.52	2.38	(47.94, 56.92)	0.61%	−3.77

Table 9. Comparison of models for FDX using the historical period January 2023 to December 2023.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.341	9.49%	28.33	241.53	22.49	(199.86, 289.21)	−1.75%	−2.59
	Heston	0.341	9.63%	28.76	239.46	22.15	(201.42, 283.63)	−2.59%	−4.66
	MJD	0.341	9.30%	27.77	240.46	22.12	(200.16, 284.99)	−2.18%	−3.66
	SVJ	0.341	**9.26%**	27.13	241.09	23.63	(198.31, 289.85)	−1.92%	−3.03
6-M	GBM	0.2759	8.19%	24.70	245.70	18.88	(212.25, 286.54)	−0.05%	1.58
	Heston	0.2759	**8.06%**	24.16	246.20	18.48	(206.53, 280.81)	0.15%	2.07
	MJD	0.2759	8.33%	24.92	247.95	19.15	(214.48, 290.22)	0.86%	3.82
	SVJ	0.2759	8.21%	24.59	245.67	18.95	(211.57, 284.37)	−0.06%	1.55
1-Y	GBM	0.2765	9.66%	28.24	256.40	20.31	(217.59, 296.25)	4.30%	12.27
	Heston	0.2765	9.68%	28.22	257.00	20.06	(219.81, 297.56)	4.55%	12.87
	MJD	0.2765	9.56%	27.98	255.50	20.86	(218.98, 298.92)	3.94%	11.38
	SVJ	0.2765	**9.45%**	27.70	255.05	20.21	(217.73, 295.22)	3.75%	10.92

Across all stocks and estimation periods (3-month, 6-month, and 1-year), the SVJ model consistently exhibits superior predictive performance when compared to the GBM, Heston, and MJD models. Specifically, the SVJ model achieves the lowest MAPE, RMSE, and absolute differences from actual prices, indicating a strong capability to capture the complex dynamics associated with high-volatility stocks.

For TSLA and MRNA, which display the highest levels of volatility among the selected assets, the SVJ model markedly outperforms the alternatives, demonstrating its strength in accounting for both stochastic volatility and abrupt price jumps. In contrast, the GBM model—limited by its assumption of constant volatility and lack of jump components—shows the weakest performance across all metrics. The Heston and MJD models offer moderate improvements, as they incorporate either stochastic variance (Heston) or jump processes (MJD), but they still fall short in capturing the full extent of price variability present in these stocks.

For relatively less volatile stocks within this group, such as GOOGL and AMZN, the performance differences across models are less pronounced, yet the SVJ model continues to yield the most accurate forecasts. META exhibits moderate volatility, and while forecast errors remain somewhat elevated across all models, the SVJ model again produces the lowest error values and the narrowest confidence intervals.

The findings provide valuable insights for financial analysts and investors by demonstrating how model selection and historical data length significantly impact stock price forecasting accuracy. Our analysis reveals that the SVJ model consistently delivers superior performance compared to GBM, Heston, and MJD models across different volatility regimes. For both relatively stable stocks and highly volatile stocks, the SVJ model achieves the lowest prediction errors, as measured by MAPE and RMSE, highlighting its effectiveness in capturing market dynamics.

The length of the historical data window emerges as another crucial factor influencing forecast quality. Shorter estimation periods (three months) tend to adapt more quickly to recent market trends but may amplify noise, while longer windows (one year) provide greater stability at the cost of reduced responsiveness to structural breaks. Our results suggest that intermediate horizons (six months) often represent an optimal compromise, particularly when using the SVJ model for highly volatile stocks.

Table 10. Comparison of models for TSLA using the historical period January 2023 to December 2023.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.4585	30.46%	63.60	246.67	31.63	(191.75, 302.95)	−0.71%	51.29
	Heston	0.4585	30.43%	63.20	248.12	32.74	(196.98, 316.56)	−0.12%	52.74
	MJD	0.4585	30.22%	63.20	247.66	34.10	(196.95, 297.59)	−0.31%	52.28
	SVJ	0.4127	**29.51%**	61.42	227.69	30.45	(168.01, 284.17)	1.66%	51.86
6-M	GBM	0.4779	30.07%	62.95	243.85	32.17	(191.03, 308.89)	−1.84%	48.47
	Heston	0.4779	29.35%	61.45	245.16	32.88	(188.92, 313.75)	−1.31%	49.79
	MJD	0.4779	29.73%	62.11	245.82	32.09	(186.58, 318.90)	−1.05%	50.44
	SVJ	0.4779	**29.08%**	60.54	247.30	33.69	(195.49, 316.51)	−0.45%	51.93
1-Y	GBM	0.5239	42.14%	87.59	272.45	42.77	(200.27, 355.19)	9.67%	77.07
	Heston	0.5239	41.22%	85.86	265.32	35.09	(207.34, 350.94)	6.80%	69.95
	MJD	0.5239	45.98%	94.90	281.17	44.38	(214.65, 368.60)	13.18%	85.80
	SVJ	0.5239	**40.90%**	83.05	273.71	42.83	(197.20, 367.92)	10.18%	78.33

Table 11. Comparison of models for GOOGL using the historical period January 2023 to December 2023.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.2964	8.49%	14.62	140.39	11.48	(118.90, 164.56)	2.22%	−1.81
	Heston	0.2964	8.46%	14.29	139.26	11.32	(119.68, 161.62)	1.40%	−2.94
	MJD	0.2964	8.15%	13.75	139.85	11.27	(119.18, 162.46)	1.83%	−2.35
	SVJ	0.2964	**8.10%**	13.42	140.07	12.03	(117.84, 164.56)	1.98%	-2.13
6-M	GBM	0.2757	8.03%	13.64	141.44	10.95	(122.01, 165.16)	2.98%	−0.77
	Heston	0.2757	**7.79%**	13.19	141.71	10.71	(118.78, 161.75)	3.18%	−0.49
	MJD	0.2757	8.05%	13.69	142.73	11.10	(123.30, 167.26)	3.92%	0.53
	SVJ	0.2757	8.01%	13.56	141.42	10.99	(121.59, 163.89)	2.96%	−0.79
1-Y	GBM	0.3045	9.18%	15.51	144.49	12.63	(120.53, 169.44)	5.20%	2.29
	Heston	0.3045	9.07%	15.35	144.93	12.50	(122.02, 170.39)	5.52%	2.72
	MJD	0.3045	9.19%	15.48	143.94	12.98	(121.38, 171.18)	4.80%	1.74
	SVJ	0.3045	**9.01%**	15.27	143.72	12.56	(120.72, 168.86)	4.64%	1.52

4.2. Case studies with extended historical time frames

To evaluate the robustness of the proposed modeling framework, parameter estimation was conducted using historical market data across three distinct calibration windows: 3 months, 6 months,

Table 12. Comparison of models for MRNA using the historical period January 2023 to December 2023.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.5321	18.18%	20.96	110.90	16.18	(82.40, 146.80)	−1.43%	10.10
	Heston	0.5321	17.82%	20.63	109.68	15.87	(83.69, 142.52)	−2.51%	8.88
	MJD	0.5321	17.53%	20.26	110.12	15.98	(82.40, 143.32)	−2.12%	9.32
	SVJ	0.5321	**17.24%**	20.18	110.93	17.07	(82.07, 148.24)	−1.39%	10.13
6-M	GBM	0.4484	14.28%	16.70	95.27	13.09	(73.55, 124.93)	−5.32%	4.55
	Heston	0.4484	13.94%	16.61	95.90	12.83	(70.29, 121.06)	−4.75%	5.18
	MJD	0.4484	14.74%	17.13	96.81	13.45	(74.86, 127.91)	−3.95%	6.09
	SVJ	0.4484	**13.76%**	16.47	95.60	13.21	(74.06, 124.42)	−5.02%	4.88
1-Y	GBM	0.4525	14.93%	17.45	104.33	13.12	(80.29, 131.11)	−7.26%	3.53
	Heston	0.4525	14.75%	17.23	104.99	13.07	(82.14, 132.60)	−6.68%	4.19
	MJD	0.4525	14.90%	17.41	103.78	13.50	(81.11, 133.00)	−7.75%	2.99
	SVJ	0.4525	**14.58%**	17.12	103.74	12.99	(80.83, 130.56)	−7.79%	2.94

Table 13. Comparison of models for META using the historical period January 2023 to December 2023.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.2698	18.53%	98.29	370.45	27.93	(317.73, 429.15)	7.48%	−73.77
	Heston	0.2698	18.39%	98.18	367.56	27.56	(319.69, 421.67)	6.64%	−76.65
	MJD	0.2698	18.35%	98.13	369.15	27.39	(318.87, 424.07)	7.10%	−75.06
	SVJ	0.2698	**18.10%**	98.06	369.47	29.22	(314.72, 428.47)	7.20%	−74.74
6-M	GBM	0.2578	17.85%	95.51	323.63	26.13	(277.42, 380.41)	3.90%	−76.16
	Heston	0.2578	17.61%	94.38	324.33	25.54	(269.93, 372.15)	4.10%	−75.46
	MJD	0.2578	17.41%	93.36	326.73	26.50	(280.48, 385.41)	4.80%	−73.07
	SVJ	0.2578	**17.26%**	92.57	323.64	26.21	(276.52, 377.53)	3.90%	−76.15
1-Y	GBM	0.3812	18.11%	96.52	389.89	43.45	(309.36, 477.49)	13.12%	−54.33
	Heston	0.3812	17.92%	95.54	391.83	43.17	(315.25, 482.20)	13.69%	−52.38
	MJD	0.3812	18.38%	97.85	388.09	44.64	(312.23, 483.38)	12.60%	−56.13
	SVJ	0.3812	**17.25%**	95.25	387.72	43.14	(310.60, 475.62)	12.49%	−56.49

Table 14. Comparison of models for AMZN using the historical period January 2023 to December 2023.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.2919	10.94%	20.93	162.55	13.29	(137.62, 190.66)	8.42%	−4.38
	Heston	0.2919	10.42%	20.62	161.24	13.11	(138.61, 187.10)	7.54%	−5.69
	MJD	0.2919	10.17%	20.14	161.94	13.03	(138.15, 188.20)	8.01%	−4.99
	SVJ	0.2627	**9.78%**	19.66	145.94	12.52	(122.69, 171.47)	7.34%	−4.29
6-M	GBM	0.2729	10.28%	20.03	138.81	11.83	(118.00, 164.59)	2.58%	−11.42
	Heston	0.2729	9.94%	19.40	139.16	11.57	(114.65, 160.87)	2.82%	−11.07
	MJD	0.2729	9.98%	19.53	140.20	12.01	(119.37, 166.89)	3.51%	−10.03
	SVJ	0.2456	**9.74%**	18.81	124.97	10.68	(105.91, 147.06)	2.35%	−10.25
1-Y	GBM	0.3292	11.32%	22.22	160.05	15.19	(131.46, 190.23)	6.75%	−6.87
	Heston	0.3292	11.11%	21.89	160.64	15.04	(133.33, 191.48)	7.14%	−6.29
	MJD	0.3292	11.50%	22.53	159.40	15.60	(132.46, 192.33)	6.32%	−7.52
	SVJ	0.3127	**10.72%**	21.10	151.23	14.34	(125.18, 180.10)	5.87%	−7.35

and 1 year. Historical datasets were obtained for each calendar year from 2018 to 2022. For each year, model parameters were estimated based on the corresponding historical data, followed by price forecasting over a fixed 3-month prediction horizon. For example, when utilizing data from the year 2022, the 1-year calibration window spanned from 01/01/2022 to 31/12/2022, with the forecast period set from 01/01/2023 to 01/04/2023. In the case of a 6-month calibration window, the data ranged from 01/07/2022 to 31/12/2022, while maintaining the same forecast interval. This estimation and forecasting procedure was systematically repeated for each year and applied to all selected stocks.

To streamline the presentation and avoid redundancy, detailed simulation results are provided for only two representative stocks: AAPL, which exhibits relatively low volatility, and TSLA, characterized by higher volatility. The use of multiple calibration windows facilitates a systematic analysis of model sensitivity to estimation period length, while ensuring coverage of market regimes marked by elevated uncertainty and abrupt price fluctuations, such as those observed during the COVID-19 pandemic.

Tables 15 through 19 summarize the simulation outcomes for AAPL over the 2018–2022 period. Forecasting performance was assessed for four stochastic models over three parameter estimation periods: 3-month, 6-month, and 1-year intervals. Across most timeframes and years, the SVJ model consistently exhibited the lowest MAPE, underscoring its superior predictive capability. Furthermore, the findings suggest that, for lower-volatility stocks, utilizing a 1-year estimation window generally results in marginally lower MAPE values compared to shorter windows.

Analogously, Tables 20 through 24 present the corresponding results for TSLA from 2018 to 2022, showing similar patterns in forecast accuracy across the same set of stochastic models and parameter estimation intervals. The SVJ model again emerged as the most accurate across the majority of configurations, outperforming the Heston, MJD, and GBM models. For higher-volatility stocks such as TSLA, the 6-month estimation window often yielded lower MAPE values relative to the 3-month and 1-year windows, suggesting that intermediate-length calibration periods may offer a favorable balance between model responsiveness and parameter stability in the presence of pronounced market fluctuations.

Table 15. Comparison of models for AAPL using the historical period January 2022 to December 2022.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.3998	18.83%	30.48	119.93	13.28	(97.16, 148.31)	−2.87%	−25.86
	Heston	0.3998	19.22%	31.07	119.00	12.93	(95.35, 146.39)	−3.62%	−26.79
	MJD	0.3998	18.68%	30.29	120.17	13.01	(98.57, 148.03)	−2.67%	−25.63
	SVJ	0.3998	**18.20%**	30.01	119.46	13.40	(96.72, 146.24)	−3.25%	−26.34
6-M	GBM	0.3547	17.69%	28.71	121.56	12.26	(99.51, 147.63)	−1.55%	−24.24
	Heston	0.3547	17.57%	28.59	121.76	12.36	(99.32, 148.90)	−1.38%	−24.03
	MJD	0.3547	17.10%	27.83	122.56	12.15	(101.90, 148.16)	−0.74%	−23.24
	SVJ	0.3547	**17.01%**	27.15	122.01	12.51	(97.48, 146.57)	−1.19%	−23.79
1-Y	GBM	0.3207	17.43%	28.09	106.47	10.72	(87.57, 129.74)	−3.77%	−24.75
	Heston	0.3207	17.30%	27.86	106.52	10.35	(86.47, 128.61)	−3.74%	−24.71
	MJD	0.3207	17.25%	27.82	106.79	10.85	(86.73, 129.10)	−3.50%	−24.42
	SVJ	0.3207	**17.16%**	27.13	106.25	10.93	(84.92, 127.53)	−3.95%	−24.97

4.3. Model performance and implications

Interestingly, despite incorporating discontinuous price jumps, the MJD model does not consistently outperform the other stochastic models across the historical windows and stock categories examined. This observation indicates that, under a variety of market conditions, the explicit modeling of stochastic volatility plays a more critical role in reducing forecast uncertainty

Table 16. Comparison of models for AAPL using the historical period January 2021 to December 2021.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.2396	21.54%	39.07	200.31	13.72	(175.65, 229.01)	12.13%	35.12
	Heston	0.2396	20.80%	37.73	198.99	13.40	(173.19, 225.73)	11.39%	33.81
	MJD	0.2396	21.70%	39.40	200.59	13.40	(177.11, 228.06)	12.28%	35.40
	SVJ	0.2396	**20.06%**	37.24	199.41	13.87	(174.44, 226.06)	11.62%	34.22
6-M	GBM	0.2241	15.58%	28.23	190.06	12.31	(167.11, 215.55)	6.39%	24.87
	Heston	0.2241	15.57%	28.11	189.93	12.32	(166.12, 215.95)	6.32%	24.75
	MJD	0.2241	16.15%	29.24	191.10	12.15	(169.66, 216.01)	6.97%	25.91
	SVJ	0.2241	**15.19%**	27.47	190.14	12.59	(164.03, 213.60)	6.44%	24.96
1-Y	GBM	0.2514	13.53%	24.88	184.55	13.36	(160.32, 212.93)	3.31%	19.36
	Heston	0.2514	13.14%	23.91	184.41	12.92	(158.46, 211.01)	3.23%	19.22
	MJD	0.2514	13.28%	24.28	184.96	13.49	(159.34, 212.16)	3.53%	19.77
	SVJ	0.2514	**13.07%**	23.43	184.02	13.69	(156.50, 209.94)	3.01%	18.84

Table 17. Comparison of models for AAPL using the historical period January 2020 to December 2020.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.3453	12.61%	18.56	135.78	13.11	(111.53, 163.87)	7.55%	10.45
	Heston	0.3453	12.21%	18.09	134.59	12.92	(112.64, 160.36)	6.61%	9.26
	MJD	0.3453	12.23%	18.04	135.17	12.88	(111.95, 161.34)	7.07%	9.84
	SVJ	0.3453	**12.15%**	17.83	135.52	13.75	(110.54, 164.10)	7.35%	10.20
6-M	GBM	0.3971	15.29%	22.24	136.96	15.55	(110.29, 171.52)	8.49%	11.63
	Heston	0.3971	**14.46%**	21.25	137.56	15.18	(106.57, 166.92)	8.97%	12.23
	MJD	0.3971	15.07%	22.26	138.78	15.86	(111.96, 175.30)	9.93%	13.45
	SVJ	0.3971	14.48%	21.40	137.21	15.63	(110.19, 170.67)	8.69%	11.89
1-Y	GBM	0.4676	15.27%	22.56	135.02	18.21	(101.97, 172.35)	6.96%	9.70
	Heston	0.4676	15.37%	22.71	135.97	18.18	(104.60, 174.70)	7.71%	10.64
	MJD	0.4676	15.10%	22.30	134.29	18.73	(103.04, 174.96)	6.37%	8.96
	SVJ	0.4676	**14.92%**	22.13	134.25	18.05	(102.69, 171.72)	6.34%	8.92

Table 18. Comparison of models for AAPL using the historical period January 2019 to December 2019.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.1802	18.66%	15.31	82.86	4.28	(75.05, 91.66)	14.10%	11.64
	Heston	0.1802	**18.05%**	14.83	82.36	4.19	(74.12, 90.48)	13.41%	11.14
	MJD	0.1802	18.76%	15.40	82.95	4.18	(75.54, 91.33)	14.23%	11.73
	SVJ	0.1802	18.27%	14.98	82.47	4.34	(74.40, 90.68)	13.57%	11.25
6-M	GBM	0.2274	15.11%	12.53	79.43	5.26	(69.65, 90.35)	9.38%	8.21
	Heston	0.2274	**15.09%**	12.47	79.38	5.26	(69.24, 90.51)	9.31%	8.16
	MJD	0.2274	15.58%	12.88	79.88	5.19	(70.72, 90.49)	9.99%	8.66
	SVJ	0.2274	15.28%	12.62	79.47	5.37	(68.36, 89.47)	9.43%	8.25
1-Y	GBM	0.2635	14.60%	12.92	78.11	5.99	(67.27, 90.92)	7.56%	6.89
	Heston	0.2635	**14.37%**	11.93	78.07	5.79	(66.50, 90.00)	7.50%	6.84
	MJD	0.2635	14.57%	12.06	78.29	6.05	(66.84, 90.52)	7.81%	7.07
	SVJ	0.2635	14.46%	11.95	77.89	6.13	(65.67, 89.58)	7.26%	6.67

than the inclusion of jump components alone. Among the models evaluated, the SVJ model demonstrates superior predictive accuracy relative to the GBM, Heston, and MJD models. This

Table 19. Comparison of models for AAPL using the historical period January 2018 to December 2018.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.4088	23.33%	11.42	31.62	3.38	(25.46, 38.77)	−15.93%	−8.90
	Heston	0.4088	22.91%	10.66	31.33	3.32	(25.62, 38.11)	−16.70%	−9.19
	MJD	0.4088	22.75%	10.58	31.46	3.34	(25.37, 38.13)	−16.38%	−9.06
	SVJ	0.4088	**22.50%**	10.48	31.59	3.57	(25.32, 39.15)	−16.02%	−8.93
6-M	GBM	0.3295	13.96%	6.71	35.76	3.24	(30.11, 42.83)	−4.95%	−4.76
	Heston	0.3295	**13.60%**	6.56	35.87	3.17	(29.16, 41.87)	−4.64%	−4.65
	MJD	0.3295	13.46%	6.49	36.14	3.30	(30.48, 43.50)	−3.92%	−4.38
	SVJ	0.3295	13.87%	6.67	35.78	3.26	(30.08, 42.55)	−4.88%	−4.74
1-Y	GBM	0.2882	12.76%	6.72	37.11	3.02	(31.35, 43.06)	−1.35%	−3.41
	Heston	0.2882	**11.54%**	5.62	37.21	2.98	(31.72, 43.22)	−1.10%	−3.31
	MJD	0.2882	12.01%	5.84	36.98	3.10	(31.55, 43.44)	−1.70%	−3.54
	SVJ	0.2882	11.90%	5.79	36.92	3.00	(31.38, 42.90)	−1.87%	−3.60

Table 20. Comparison of models for TSLA using the historical period January 2022 to December 2022.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.6963	54.85%	105.27	75.82	13.50	(55.12, 108.82)	−29.86%	−98.63
	Heston	0.6963	54.65%	104.89	76.17	14.74	(51.61, 110.32)	−29.53%	−98.27
	MJD	0.6963	54.58%	104.84	76.86	17.40	(55.91, 106.77)	−28.90%	−97.59
	SVJ	0.6963	**52.43%**	101.32	80.06	15.06	(54.86, 109.57)	−25.94%	−94.39
6-M	GBM	0.5905	44.96%	86.79	91.29	15.72	(64.18, 122.74)	−12.55%	−77.93
	Heston	0.5905	43.89%	86.33	89.76	15.58	(64.63, 126.25)	−13.97%	−79.45
	MJD	0.5905	43.92%	86.47	89.35	13.27	(66.81, 116.16)	−14.35%	−79.86
	SVJ	0.5905	**43.82%**	85.91	88.09	15.98	(59.94, 117.62)	−15.52%	−81.13
1-Y	GBM	0.6710	44.70%	88.25	93.41	15.23	(68.32, 127.41)	−13.59%	−81.04
	Heston	0.6710	43.84%	86.77	94.94	16.08	(64.13, 126.64)	−12.17%	−79.50
	MJD	0.6710	44.05%	87.09	94.99	17.33	(64.44, 129.56)	−12.13%	−79.46
	SVJ	0.6710	**43.71%**	86.16	96.48	21.74	(64.42, 155.12)	−10.75%	−77.97

Table 21. Comparison of models for TSLA using the historical period January 2021 to December 2021.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.6068	55.43%	185.50	466.57	84.57	(336.88, 681.52)	16.66%	155.10
	Heston	0.6068	**53.42%**	178.96	468.88	91.82	(318.42, 672.74)	17.24%	157.41
	MJD	0.6068	53.77%	178.96	471.31	104.71	(344.15, 646.30)	17.85%	159.85
	SVJ	0.5461	54.29%	181.06	442.49	83.60	(299.34, 611.10)	20.65%	162.18
6-M	GBM	0.4062	39.09%	127.60	380.38	54.95	(282.72, 486.78)	10.12%	115.63
	Heston	0.4062	37.26%	122.56	374.94	54.15	(285.98, 500.63)	8.76%	110.19
	MJD	0.4062	37.09%	121.74	374.31	46.56	(293.75, 466.34)	8.59%	109.57
	SVJ	0.4062	**35.77%**	117.31	368.70	55.23	(266.96, 466.87)	7.19%	103.95
1-Y	GBM	0.5430	38.72%	127.82	413.06	56.89	(317.75, 539.18)	3.28%	101.60
	Heston	0.5430	38.08%	125.59	418.90	60.63	(300.47, 534.91)	4.75%	107.44
	MJD	0.5430	37.97%	125.97	419.42	64.71	(300.77, 544.50)	4.87%	107.95
	SVJ	0.5159	**37.23%**	125.55	402.69	75.59	(284.62, 601.72)	5.69%	106.79

advantage is particularly evident in highly volatile stocks, where the SVJ model consistently attains the lowest RMSE and MAPE across all historical estimation periods. These results suggest that

Table 22. Comparison of models for TSLA using the historical period January 2020 to December 2020.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.5790	32.51%	92.98	308.25	53.27	(219.22, 409.68)	26.72%	57.19
	Heston	0.5790	31.23%	90.68	311.05	55.15	(229.02, 428.53)	27.87%	59.98
	MJD	0.5790	30.66%	89.93	310.05	58.40	(228.20, 395.89)	27.46%	58.99
	SVJ	0.5211	**30.66%**	90.17	287.69	52.37	(191.99, 384.04)	28.26%	61.73
6-M	GBM	0.8037	40.34%	116.52	321.52	77.25	(204.82, 492.87)	32.17%	70.45
	Heston	0.8037	**38.88%**	114.41	325.11	80.23	(203.42, 509.67)	33.65%	74.05
	MJD	0.8037	39.18%	115.89	325.72	76.86	(196.54, 516.49)	33.90%	74.66
	SVJ	0.7716	39.68%	119.00	317.99	79.93	(206.28, 498.51)	34.72%	76.97
1-Y	GBM	0.8969	37.90%	113.03	311.54	88.04	(179.86, 499.01)	28.07%	60.47
	Heston	0.8969	**30.78%**	90.78	296.05	71.48	(191.35, 479.27)	21.70%	44.99
	MJD	0.8969	41.40%	122.69	329.52	101.03	(201.50, 524.47)	35.46%	78.46
	SVJ	0.8072	35.28%	104.47	283.53	79.02	(157.90, 469.94)	26.56%	57.57

Table 23. Comparison of models for TSLA using the historical period January 2019 to December 2019.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.4792	27.95%	13.98	37.58	5.50	(28.84, 51.19)	31.00%	−3.88
	Heston	0.4792	26.53%	13.93	37.67	5.96	(27.59, 50.42)	31.33%	−3.78
	MJD	0.4792	26.35%	13.93	37.84	6.59	(29.44, 48.92)	31.90%	−3.62
	SVJ	0.4600	**25.99%**	13.35	37.57	5.71	(27.44, 48.83)	34.98%	−2.23
6-M	GBM	0.4901	26.55%	14.93	33.35	4.98	(24.52, 43.00)	16.26%	−8.11
	Heston	0.4901	25.96%	14.47	32.86	4.92	(24.84, 44.34)	14.57%	−8.59
	MJD	0.4901	25.30%	14.18	32.80	4.22	(25.53, 41.16)	14.35%	−8.65
	SVJ	0.4705	**25.29%**	14.00	31.00	4.80	(22.20, 39.51)	12.09%	−8.79
1-Y	GBM	0.4958	29.08%	16.22	28.98	3.63	(22.84, 36.93)	1.03%	−12.47
	Heston	0.4958	28.68%	16.02	29.33	3.86	(21.69, 36.65)	2.26%	−12.12
	MJD	0.4958	28.95%	16.11	29.37	4.10	(21.73, 37.28)	2.40%	−12.08
	SVJ	0.4760	**28.19%**	15.51	28.44	4.82	(20.76, 40.92)	3.17%	−11.36

Table 24. Comparison of models for TSLA using the historical period January 2018 to December 2018.

Period	Model	σ	MAPE	RMSE	Mean Price	Std. Dev.	95% CI	% Change	Diff. from Actual
3-M	GBM	0.6690	20.59%	4.88	23.53	4.05	(16.76, 31.03)	3.82%	1.45
	Heston	0.6082	19.28%	4.52	21.59	3.85	(15.83, 29.84)	4.44%	1.52
	MJD	0.6082	19.03%	4.53	21.52	4.07	(15.79, 27.42)	4.07%	1.44
	SVJ	0.5474	**18.29%**	4.35	19.96	3.63	(13.28, 26.78)	6.56%	1.90
6-M	GBM	0.7416	20.53%	4.88	22.52	4.20	(15.94, 31.39)	−1.08%	0.44
	Heston	0.6742	19.04%	4.52	20.67	3.97	(14.30, 29.31)	−0.04%	0.59
	MJD	0.6742	18.60%	4.40	20.71	3.83	(14.03, 29.81)	0.15%	0.63
	SVJ	0.6405	**18.29%**	4.18	19.88	3.88	(14.24, 28.20)	1.18%	0.82
1-Y	GBM	0.6962	20.57%	4.87	24.67	4.19	(17.72, 32.82)	-0.70%	0.58
	Heston	0.6962	**17.58%**	4.18	23.99	3.44	(18.41, 32.45)	−4.01%	−0.11
	MJD	0.6962	20.40%	4.84	25.52	4.39	(19.02, 34.20)	3.47%	1.44
	SVJ	0.6266	19.02%	4.48	22.34	3.78	(15.68, 30.77)	0.01%	0.65

capturing both volatility clustering and abrupt price discontinuities yields notable improvements in forecasting precision under turbulent market regimes.

In the context of less volatile stocks, the SVJ model also marginally outperforms the competing models across almost all historical periods, although the magnitude of improvement is comparatively smaller. These findings suggests that while the inclusion of jump dynamics offers more substantial gains in predictive accuracy for high-volatility assets, the SVJ model maintains robust performance across a broad spectrum of volatility profiles. From a practical perspective, the Heston model may remain attractive to financial practitioners due to its parsimonious structure and competitive forecasting performance, especially in scenarios where computational efficiency is paramount. However, for applications where high-fidelity forecasts are essential—such as risk management under extreme market conditions or pricing derivatives in volatile environments—the SVJ model is more suitable.

Overall, the results underscore that model selection should be informed not only by the asset's inherent volatility characteristics but also by the desired forecast horizon and the acceptable level of model complexity. For relatively stable stocks such as AAPL, MSFT, IBM, WMT, and FDX, the SVJ model with a 1-year calibration window provides an effective compromise between predictive accuracy and temporal stability. Conversely, for more volatile stocks such as TSLA, GOOGL, MRNA, META, and AMZN, the SVJ model offers marked forecasting advantages, particularly when calibrated over shorter windows—such as 6 months—to adequately capture rapid shifts in market dynamics.

4.4. Limitations and future research

Despite the promising results, several limitations must be acknowledged. First, model evaluation was conducted using historical data, which may not capture the full range of potential future market scenarios, particularly under regime shifts. Second, the models assume stationary parameters over the estimation and prediction horizons, an assumption that may be violated in real-world markets characterized by structural breaks and evolving investor behavior.

Future research could address these limitations by incorporating models with time-varying parameters, regime-switching dynamics, or machine learning techniques that allow for adaptive calibration. Additionally, exploring hybrid approaches that integrate macroeconomic indicators or market sentiment measures into the SVJ framework may further enhance predictive performance. Nonetheless, the present study contributes valuable empirical insights into the relative performance of classical and advanced stochastic models, offering a solid foundation for both practical implementation and future methodological developments in financial forecasting.

5. Conclusions

This study systematically evaluated the forecasting performance of four prominent stochastic models—GBM, Heston, MJD, and SVJ—across a diverse set of stocks and varying historical periods. By evaluating predictive accuracy over a three-month horizon using various historical data periods, we assess the efficacy of these models in capturing the intricate dynamics of stock price behavior.

The empirical results demonstrate that the SVJ model consistently outperforms the other models in terms of RMSE and MAPE across a wide range of market conditions. Its ability to jointly capture stochastic volatility and discontinuous price movements renders it particularly effective in high-volatility environments, as evidenced by its superior performance for stocks such as TSLA and MRNA. For less volatile stocks like AAPL and MSFT, while the performance gains of the SVJ model are more modest, it still provides the lowest forecast errors across all calibration windows.

Furthermore, the findings highlight the importance of tailoring the estimation window to the asset's volatility profile. A 1-year calibration period offers improved stability and accuracy for less volatile stocks, whereas a 6-month window proves more responsive to market dynamics for highly volatile assets. Although the Heston model remains a practical choice for its parsimony and computational efficiency, the SVJ model is better suited for applications that demand high predictive fidelity.

Author contributions

The first author was responsible for conceptualization, formal analysis, investigation, and methodology. The second author contributed to supervision, visualization, and writing—review and editing.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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