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Examining the Intermittency in the Swing Equation

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Abstract: Studying the nonlinear dynamical systems and their stability is important for various engineering applications, especially with power systems. While previous studies have examined primary, subharmonic resonances and quasiperiodicity in nonlinear systems, the phenomena of intermittency remain unfamiliar. This study analyses intermittency in the swing equation, which is a second-order differential equation that characterises the dynamic behaviour in power systems. Intermittency, modelled by sudden bursts within periodic regions, plays a vital role in the transition from stability to chaos. It also identifies the conditions under which intermittency occurs, mainly when varying the inertia and voltage of the machine. Numerical simulations, bifurcation diagrams, Poincaré maps, heat maps and Lyapunov exponents are used to determine intermittency. Findings show that intermittency happens as a precursor to chaos, affecting the stability of the system. Results also indicate small disturbances can induce instability, thereby providing insights into the control aspect. It contributes to a broader understanding of the swing equation and highlights the importance of identifying the precursors to chaos to mitigate the adverse effects.

Key-Words: nonlinear dynamics; swing equation; intermittency; power system

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1 Introduction

The swing equation, a second-order differential equation, plays an important role in understanding the dynamical behaviour within power systems [1]. It is widely used in engineering applications, especially in the electrical engineering sector, to model the stability of synchronous machines [2, 3]. Although extensive research have been conducted with regards to primary, subharmonic resonances and quasiperiodicty, the phenomena of intermittency remains under explored in swing equation. Sudden fluctuations or bursts within a periodic pattern in a nonlinear system is called intermittency [4]. This study aims to fill this gap by systematically analysing how intermittency occurs and how it contributes to the system moving from stable regions to chaotic regions [5]. Different types of this phenomenon, such as Type I, Type II and Type III have been identified in experimental settings, implying its practical implications [6, 7]. Type I intermittency is often associated with saddle-node bifurcation and Type II is related to subcritical hoph bifurcation.

Recent studies in nonlinear systems have shown that intermittency can serve as a critical precursor to chaos, thereby affecting the stability of the systems. For instance, intermittency has been observed in fluid dynamics, thermoacoustic oscillations and electrical circuits where small parameter alterations lead to sudden disruptions [8]. Within the context of power systems, it is vital to understand the intermittency due to the increasing integration of renewable energy sources, which introduce low inertia and greater system variability [9]. The observed phenomena of intermittency can have a significant indication of unstable power grids, making it important to study it within the swing equation framework [10]. Despite the relevance of this occurrence, current research on nonlinear power systems overlooks intermittency as a clear route to chaos. Previous works of literature have analysed and studied period-doubling, quasiperiodcity and crisis, only a few studies have investigated intermittency when varying parameters such as inertia and voltage of the machine.

The main objective of this research is to examine the role of intermittency in the swing equation and its effects on the stability of the power systems. The study aims to investigate the occurrence of intermittency when parameters are varied, and examine the transition from periodic to chaotic regions while identifying critical points for the emergence of intermittency. The initial focus is also on the influence of inertia and voltage of the machine to study the stability of the swing equation.

To achieve these aims and objectives, this paper employs analytical and numerical techniques. Fourth-order Runge Kutta method is used to solve the swing equation in Matlab. Bifurcation diagrams are obtained to analyse stability transitions, while the heat maps show insights into the evolution of system dynamics. Lyapunov exponents are also calculated to validate the presence of chaos and intermittency. Poincaré maps are also used to study the system's trajectories. This research provides a comprehensive understanding of the behaviour of the system when inertia and voltage of the machine are changed.

The results have significant implication for designing better stability and control strategies, especially in power circuits that undergo sudden fluctuation. Mitigating intermittent behaviour at early stage can prevent adverse effects, ensuring a reliable nonlinear power system.

1.1 Brief Literature Review

The stability of nonlinear power systems is influenced by multiple phenomena, including bifurcations, chaos and intermittency. Various studies have examined the stability of these systems using analytical and numerical techniques such as bifurcation diagrams, Lyapunov exponents and phase space analysis [1, 11, 12, 13].

Bifurcation theory plays an important part in understanding how nonlinear systems transition between stable and unstable states [14]. Period-doubling bifurcation in particular is a well known route to chaos as shown in power systems and electrical circuits [1, 2]. The bifurcation diagrams are also used to demonstrate the shifts by plotting systems states against a control parameter, revealing regions of stability and intermittency [15].

Lyapunov exponent is a key indicator of chaotic behaviour, which measures close trajectories in a system. A positive Lyapunov exponent exemplifies chaotic motion while a negative exponent indicates stability [11, 12]. This technique has been used vastly to study stability and dynamical behaviour in nonlinear systems. The swing equation's stability can be analysed using Lyapunov exponents, especially intermittent phenomena. Solving swing equation requires accurate numerical integration techniques. The fourth-order Runge Kutta method is widely used to solve non-linear differential equations, providing high accuracy in computing systems [1, 2]. Poincaré maps offer geometric illustrations of periodic and chaotic attractors, aiding to visualise the intermittent regions [12, 16]. Another route to quasiperiodicity, where the system has irrational frequency values causing irregular transitions [17].

Unlike gradual transitions observed in period-doubling intermittency is characterised by sudden bursts of instability within a periodic system [4]. Intermittency can be seen in fluid dynamics, thermoacoustic oscillations [6] and combustion instability [10]. Studies have also shown that intermittency can begin when critical system parameters such as inertia, damping and voltage fluctuate beyond stability thresholds [18, 19].

There are three types of intermittencies. Type I intermittency is where system trajectories remain near an unstable fixed point before sudden bursts occur [7]. Type II intermittency is often observed in electrical and mechanical systems [6]. Type III intermittency is related to quasiperiodicity and happens due to non-uniform reinjection in probability densities [8].

Research also found that the stability of the swing equation can also be studied by modelling it on Matlab Simulink. It helps with visualising the system better and provides a comprehensive understanding of the dynamical behaviour [20].

When it comes to the analysis of the dynamics of a power system, the swing equation, which is investigated in this research effort, is an extremely important component [21]. It does display features that are comparable to those of other power systems; however, it is essential to conduct an in-depth analysis of it first in order to acquire a deeper comprehension of the ideas. It has been discovered that the generalised form of the swing equation is also beneficial in terms of comprehending the concept of transient stability in power-electronic power systems [22]. In response to even the slightest disturbance, the rotor of the machine will exhibit some motion in relation to the air gap that is revolving in a synchronised manner. Following this, a relative motion is initiated, which enables the swing equation to be utilised for the purpose of describing and modelling this relative motion [23, 24].

2 Methodology

The swing equation studied here depicts the motion of rotor of machine as reproduced below as Figure 1 [1, 3, 11, 12, 18, 19].

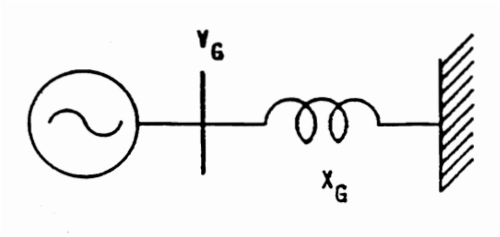

Fig. 1: Swing equation describing the motion of the rotor of the machine. Figure reproduced from [1].

The equation analysing the rotor's motion of the machine including a damping term is given by [1, 3].

$$\frac{2H}{\omega_R}\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} = P_m - \frac{V_G V_B}{X_G}\sin(\theta - \theta_B) \tag{1}$$

$$V_{\mathrm{B}} = V_{B0} + V_{\mathrm{B1}}\cos(\Omega t + \phi_v) \tag{2}$$

$$\theta_B = \theta_{B0} + \theta_{B1}\cos(\Omega t + \phi_0) \tag{3}$$

with
$\omega_R = Constant\ angular\ velocity$,
H= Inertia,
D= Damping,
P_m= Mechanical Power,
V_G= Voltage of machine,
X_G = Transient Reactance,
V_B= Voltage of busbar system,
θ_B = phase of busbar system,

V_{B1} and θ_{B1} magnitudes are assumed to be small.

2.1 Analytical Work

Initially the equilibrium points are found. At equilibrium, the time derivatives of θ vanishes giving:

$$\frac{d\theta}{dt} = 0, \quad \frac{d^2\theta}{dt^2} = 0$$

Substituting these into the swing equation:

$$0 = P_m - \frac{V_G V_B}{X_G}\sin(\theta_0 - \theta_B)$$

Rearranging,

$$P_m = \frac{V_G V_B}{X_G}\sin(\theta_0 - \theta_B)$$

where θ_0 is the equilibrium rotor angle.

Linearising the swing equation system by introducing a small perturbation. Initially, defining a small perturbation $\delta = \theta - \theta_0$, where δ represents small deviations from equilibrium. Using a Taylor series expansion for $\sin(\theta - \theta_B)$ about θ_0,

$$\sin(\theta - \theta_B) \approx \sin(\theta_0 - \theta_B) + (\theta - \theta_0)\cos(\theta_0 - \theta_B)$$

Since

$$P_m = \frac{V_G V_B}{X_G}\sin(\theta_0 - \theta_B),$$

substituting into the swing equation gives:

$$\frac{2H}{\omega_R}\frac{d^2(\delta)}{dt^2} + D\frac{d(\delta)}{dt} = -\frac{V_G V_B}{X_G}\cos(\theta_0 - \theta_B)\delta$$

Rearranging,

$$\frac{d^2\delta}{dt^2} + \frac{D}{2H}\frac{d\delta}{dt} + \frac{V_G V_B}{2HX_G}\cos(\theta_0 - \theta_B)\delta = 0$$

This is a linearised second-order differential equation of the form:

$$\ddot{\delta} + \alpha\dot{\delta} + \beta\delta = 0$$

where:

$$\alpha = \frac{D}{2H}, \quad \beta = \frac{V_G V_B}{2HX_G}\cos(\theta_0 - \theta_B)$$

The characteristic equation of this system is:

$$\lambda^2 + \alpha\lambda + \beta = 0$$

Solving for λ using the quadratic formula:

$$\lambda = \frac{-\alpha \pm \sqrt{\alpha^2 - 4\beta}}{2}$$

The nature of the eigenvalues determines stability:

Stable system is when both roots have negative real parts ($\mathrm{Re}(\lambda) < 0$), then the system returns to equilibrium. This occurs when:

$$\alpha^2 - 4\beta > 0, \quad \text{and} \quad \alpha > 0$$

Marginal stability is when at least one root is zero ($\lambda = 0$), the system is neutrally stable (quasiperiodic motion).

An unstable system occurs if any root has a positive real part ($\mathrm{Re}(\lambda) > 0$), then small perturbations grow exponentially, leading to bifurcations and chaotic behavior. This occurs when:

$$\alpha^2 - 4\beta < 0, \quad \text{or} \quad \alpha < 0$$

2.2 Lyapunov Exponent Analysis

The Lyapunov exponent λ quantifies the system's sensitivity to initial conditions by measuring the average exponential rate of divergence of nearby trajectories in phase space [25, 26, 27]. It is computed as:

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \ln \left| \frac{\delta\theta(t)}{\delta\theta(0)} \right| \tag{1}$$

where $\delta\theta(t)$ represents the perturbation in the rotor angle over time.

The Lyapunov exponent characterises different dynamical behaviors as stated below:

Stable periodic motion: $\lambda < 0$, indicating that small disturbances decay, and the system returns to its steady-state region [1, 5].

Intermittency: λ fluctuates between negative and positive values with sudden bursts, reflecting a system that alternates between stability and chaos [6].

Chaos: $\lambda > 0$, signifying exponential divergence of trajectories, leading to unpredictable behavior and unstable regions [7].

In our study, the system transitions from periodic motion to chaos through intermittent bursts when:

$$\lambda \approx 0, \quad \frac{d\lambda}{dr} > 0 \text{ at } r = r_c. \tag{2}$$

where r_c represents the critical value of the bifurcation parameter r at which the system shifts from stability to chaos. This transition is confirmed through the computed Lyapunov exponents, which align with the bifurcation diagrams and Poincaré maps.

2.3 Bifurcation Diagrams

The bifurcation diagrams are generated by incrementally increasing the forcing parameter r, while continuing the time integration of the system at each step [1, 3, 11]. For each value of r, the maximum amplitude of the oscillatory solution is computed and plotted against r. This process reveals how the system's behaviour evolves as the forcing parameter is varied, showing transitions between periodic, chaotic states and even intermittency. The r is considered as follows,

$$r = \frac{V_G V_B}{X_G} \sin(\theta - \theta_B).$$

Swing equation (1) was solved using the fourth-order Runge Kutta method for numerical accuracy. The inertia and voltage of the machine variables were then altered separately to observe the minute changes that occur within the nonlinear system. Primary and subharmonic resonances are in focus of this study to analyse intermittent behaviour.

Initially, bifurcation diagrams were obtained for the primary and subharmonic resonance of the swing equation. This is then validated with heat maps and Lyapunov exponents to validate the results. Then this study discusses the changes observed in the system when the inertia and voltage of the machine are altered separately. Poincaré maps were then obtained for the time series to compare the results and intermittency was seen when a slight change was made to the nonlinear system. This shows that a small disturbance can lead to complex behaviour within the system.

Intermittency is considered a route to chaos. The swing equation also exhibits intermittency when variables are altered. This study will provide a strong foundation for researchers to focus on this topic in other nonlinear systems. Previous research by the same authors [3, 11, 12, 18, 19] studied the primary, subharmonic resonances, and quasi-periodicity in the swing equation. This is a continuation of those research and will give a wider knowledge of the case of intermittency.

2.4 Graphical Representation

To investigate the intermittent behaviour, numerical simulations were conducted using fourth-order Runge Kutta method. This method was chosen due to its accuracy in solving second order differential equations. The testing procedure involved selecting parameters such as inertia and voltage of machine and incrementing slightly to observe any sudden bursts within a periodic region. Bifurcation diagrams were plotted and variation of the variable r allowed to identify stability lost.

To ensure validity and reliability different conditions were considered. The findings were cross-validated using bifurcation diagrams, Lyapunov exponents and heat maps. These figured provided insights into the stability regions and confirmed the presence of intermittency. There were limitations in this research method, including computational complexity during numerical simulations.

Analysing Intermittency around Primary Resonance

Initially, the bifurcation diagram related to $\Omega = 8.3$ rads^{-1}, which is a value closer to the primary resonance $\Omega = 8.27$ rads^{-1} is obtained for analysis as shown in Figure 2. At around $r = 0.97$, period doubling bifurcation occurs showing a stable period 2 orbit. Approximately when $r = 2.235$, another period doubling bifurcation occurs depicting a period 4 orbit, which then leads to a period 8 orbit and so on. This happens for 2^n where n takes large values. This then leads to an aperiodic motion eventually cascading to an unstable system [28, 29, 30].

At around $r = 2.82$, all periodic orbits would have occurred by then and furthermore when $r = 3.03$, there are periodic orbits of all periods and all of them are unstable.

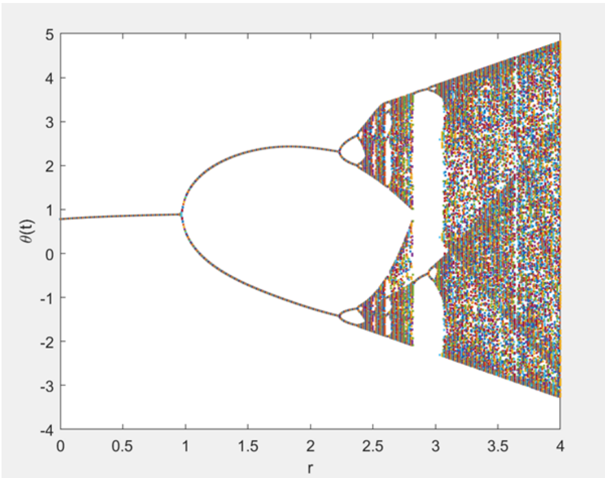

Fig. 2: Bifurcation diagram for $\Omega = 8.3$ rads^{-1}.

The following heatmap, Figure 3, shows the progression of the system into an unstable region. The output heatmap represents the dynamic behaviour of the system over a range of r values and iterations n [31]. Each pixel's colour corresponds to the 2π value of x_n, which highlights periodicity and chaos distinctly. Periodic behaviour appears as horizontal bands of uniform colour, where the system settles into repeating patterns. Chaotic regions are characterised by irregular, scattered, and mixed colour patterns, indicating unpredictable behaviour. The marked region at $r = 2.82$ demonstrates intermittency, where the system alternates between chaotic and periodic dynamics. Above $r = 3.03$, the heatmap predominantly shows chaotic behaviour, represented by noisy, non-uniform patterns. This confirms that intermittency acts as an intermediate stage before chaos emerges. The transition is further validated using bifurcation diagrams and Lyapunov exponents.

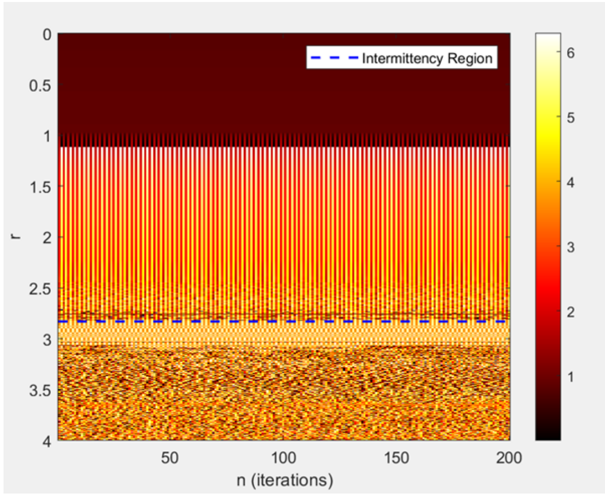

Fig. 3: Heatmap depicting the dynamical behaviour closer to Primary Resonance.

The following figure, Figure 4, shows the corresponding Lyapunov exponents for the corresponding bifurcation diagram. Just below the point of tangency, the Lyapunov exponent is positive. This shows that the dynamics just below the tangent bifurcation are actually chaotic. Once this tangency is obtained, there is a bifurcation where the Lyapunov exponent becomes zero. Then, it can be seen that the exponent values become negative and then gradually go into positive exponents. This validates the results obtained from the bifurcation diagram and the heat map.

Research also found the importance of Lyapunov Exponents in validating the detailed investigation of bifurcation diagrams [25]. Hence, this plays a vital role in studying the intricate dynamic behaviour of nonlinear systems.

The results obtained for intermittency around the primary resonance depicted intermittency. The bifurcation diagram showed the route of period doubling to chaos, with intermittency observed at $r = 2.82$. The heat map confirmed

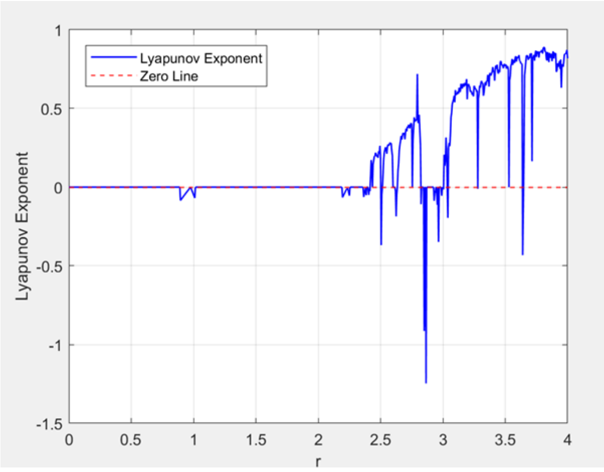

Fig. 4: Lyapunov Exponents at $\Omega = 8.3$ rads^{-1}.

this, revealing an area where periodic behaviour unpredictably changed to chaos. Lyapunov exponents also fluctuated between positive and negative values at the same r value, confirming intermittent behaviour.

Analysing Intermittency around the Subharmonic Resonance

The subharmonic resonance for the swing equation in this study is at $\Omega = 19.41$ rads^{-1}. The bifurcation diagram is obtained when $\Omega = 19.5$ rads^{-1}, a value closer to the subharmonic resonance as shown in Figure 5. At around $r = 0.965$, period doubling bifurcation occurs showing a stable period 2 orbit. Approximately when $r = 2.36$, another period-doubling bifurcation occurs depicting a period 4 orbit. The swing equation system then goes into chaos

At around $r = 2.685$, all periodic orbits would have occurred by then and furthermore when $r = 2.9$ the system becomes unstable.

The following heatmap, Figure 6, shows the system's behaviour for subharmonic resonance. The marked region at $r = 2.68$ represents intermittency, where the system alternates between chaotic and periodic dynamics. This transition is characterised by sudden bursts of instability, which momentarily disrupt the otherwise regular periodic structure. The presence of intermittency at this stage suggests that the system is in a transitional phase, where it is highly sensitive to parameter variations.

Above $r = 2.9$, the heatmap shows a fully chaotic state, as indicated by the irregular and scattered colour distribution. Unlike

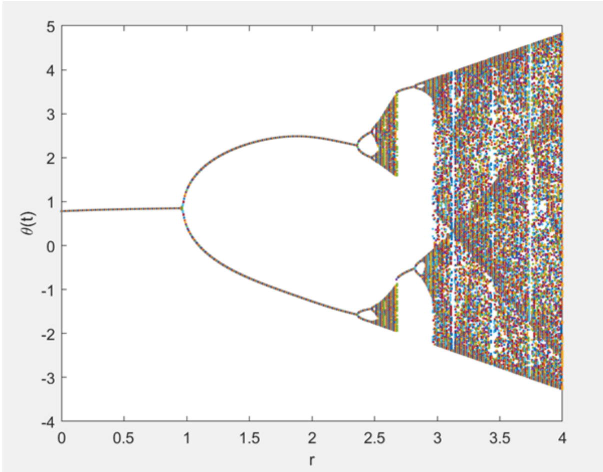

Fig. 5: Bifurcation diagram at $\Omega = 19.5$ rads^{-1}.

the structured bands observed in the periodic regions, the chaotic regime exhibits no clear pattern, signifying the loss of system stability. This behaviour is evident in the heatmap, where irregular bursts can be seen through the scattered colour patterns, marking the shift from intermittency to complete chaos.

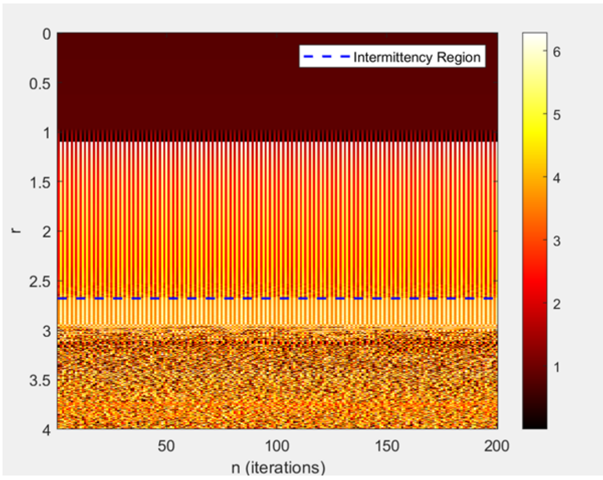

Fig. 6: Heatmap depicting the dynamical behaviour closer to Subharmonic Resonance.

Figure 7, shows the Lyapunov exponents for the corresponding bifurcation diagram for subharmonic resonance. This validates the results obtained from the bifurcation diagram and the heat map for this case. Beyond $r = 2.9$, the Lyapunov exponent remains consistently positive, signifying the complete transition to chaos. This indicates that small initial differences in the state of the system grow exponentially over time, leading to highly unpredictable dynamics. The alignment of these results with the bifurcation diagram and the heat map further strengthens the conclusion that intermittency plays a crucial role in the stability transition of the swing equation.

These findings emphasise the importance of intermittency for subharmonic resonance within the swing equation, showing sudden chaotic motion. The presence of intermittent bursts highlights the system's behaviour in small parameter variations.

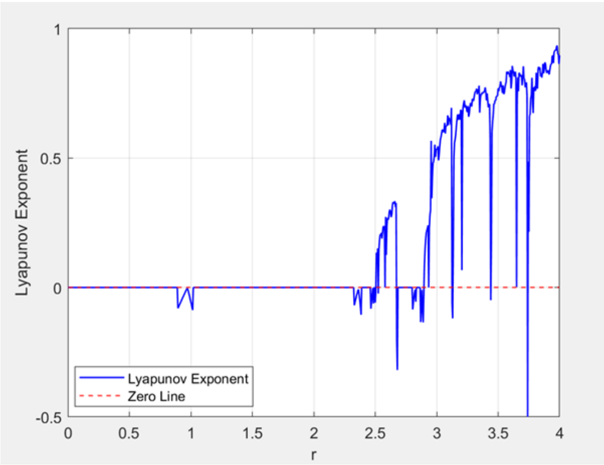

Fig. 7: Lyapunov Exponents at $\Omega = 19.5$ rads^{-1}.

Analysing Intermittency around the Quasiperiodicity

The bifurcation diagram related to the quasiperiodicity at $\Omega = \pi/2.5$ rads^{-1} is shown in Figure 8. At around $r = 2.24$, the first case of intermittency can be seen. Next at $r = 2.5$, intermittency can be observed again from the bifurcation diagram. At $r = 3.701$ sudden burst of intermittent behaviour can be observed again.

The following heatmap, Figure 9, shows the system for quasiperiodicity. The marked regions at $r = 2.24$, 2.5, 3.701 depict intermittencies, where the system alternates between chaotic and periodic dynamics. It provides insight into intermittency under quasiperiodic conditions, where the forcing frequency is set to $\Omega = \pi/2.5$ rads^{-1}. Unlike the previous cases, quasiperiodicity introduces multiple critical points where intermittency is observed.

As r increases beyond $r = 3.7$, the chaotic regions dominate the heatmap, signifying the breakdown of quasiperiodicity into full chaos. This behaviour suggests that even slight parameter fluctuations in quasiperiodic systems

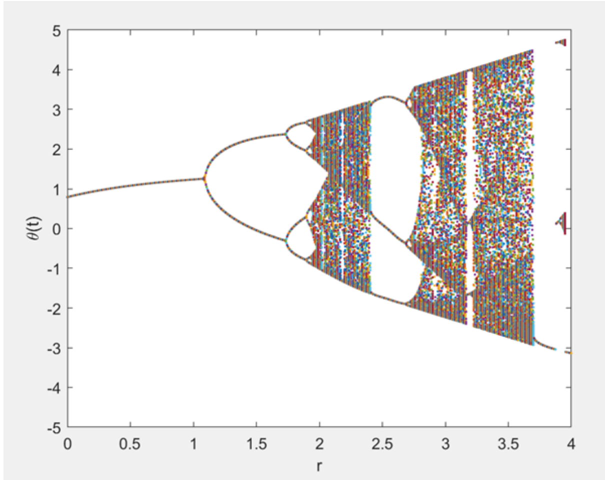

Fig. 8: Bifurcation diagram for Quasiperiodicity at $\Omega = \pi/2.5$ rads^{-1}.

can trigger intermittent instability, a crucial factor in nonlinear power system analysis.

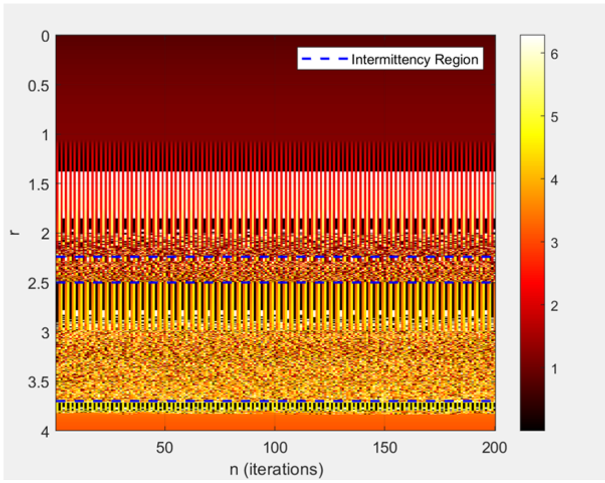

Fig. 9: Heatmap depicting the dynamical behaviour for Quasiperiodicity.

Figure 10 shows the Lyapunov exponents for the corresponding bifurcation diagram for the case of quasiperiodicity. This validates the results obtained from the bifurcation diagram and the heat map, confirming the presence of intermittency as a precursor to chaos. Initially, the Lyapunov exponent remains negative, indicating stable periodic motion. However, as r increases, the exponent fluctuates between negative and positive values, reflecting intermittent bursts of instability.

Hence it is crucial to understand the intermittency within the context of quasiperiodicity to predict instability. This will help engineers to develop more effective stability control strategies, minimising the risk of sudden system failures.

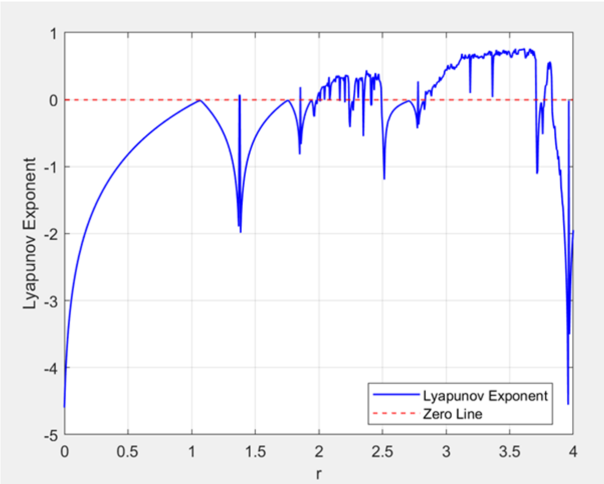

Fig. 10: Lyapunov Exponents for the case of Quasiperiodicity at $\Omega = \pi/2.5$ rads^{-1}.

Effects of altering Inertia in the Swing Equation

The following results were obtained when the inertia altered and exhibited intermittency.

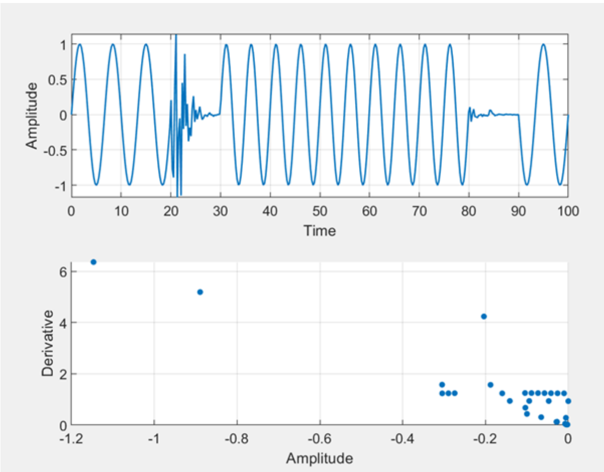

Fig. 11: Time series and Poincaré map for the case of intermittency when Inertia is 1.81 kgm^2.

The above figures, Figures 11, 12, and 13, show the time series when the variable inertia is changes. The corresponding Poincaé maps show the chaotic attractor when after periodic behaviour. For example, when H = 1.81 kgm^2, periodic orbits are obtained and sudden chaotic bursts are obtained in between. Then the

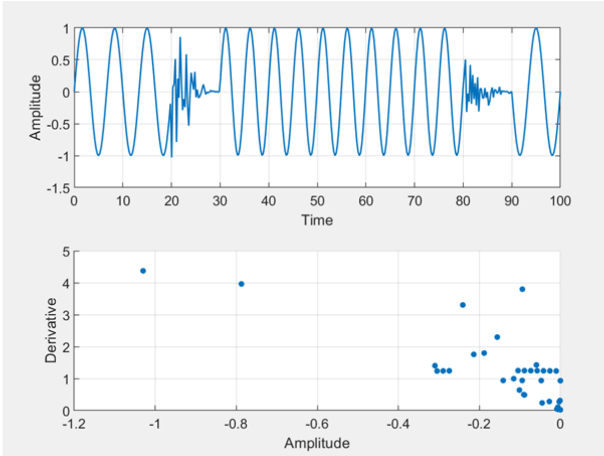

Fig. 12: Time series and Poincaré map for the case of intermittency when Inertia is 1.75 kgm^2.

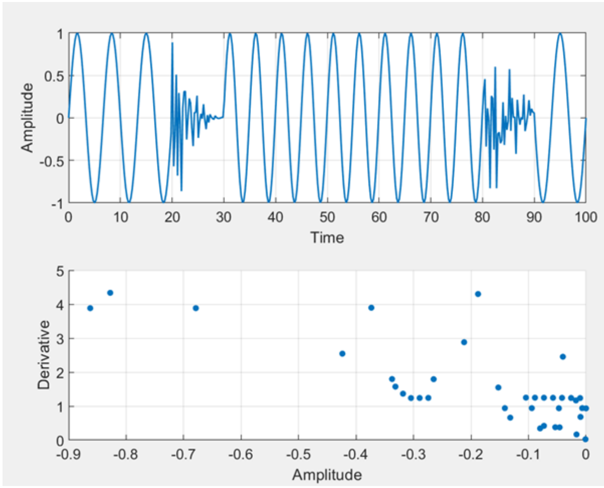

Fig. 13: Time series and Poincaré map for the case of intermittency when Inertia is 1.7 kgm^2.

Poincaré map also shows periodic attractor and then gradually unstable region can be seen.

Effects of altering Voltage of machine in the Swing Equation

The following intermittency graphs were obtained when the machine voltage (V_G) is changed.

The Figures 14, 15 and 16 all show the time series and Poincaré maps when the voltage of machine is changed. Here again it can be observed that the system goes to sudden unstable stage within the periodic behaviour, exhibiting intermittency.

The inertia and voltage of the machine

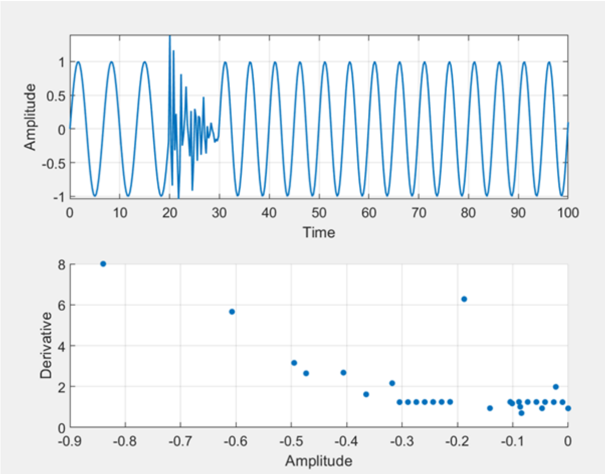

Fig. 14: Time series and Poincaré map for the case of intermittency when V_G is 0.05.

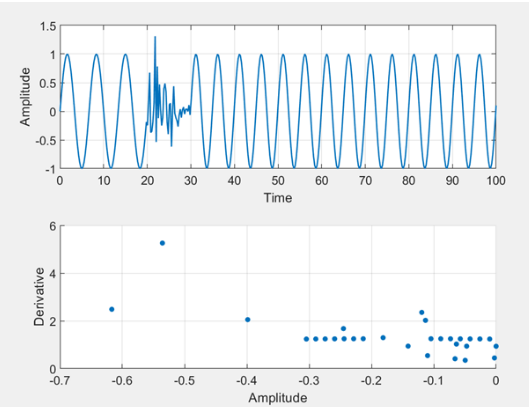

Fig. 15: Time series and Poincaré map for the case of intermittency when V_G is 0.04.

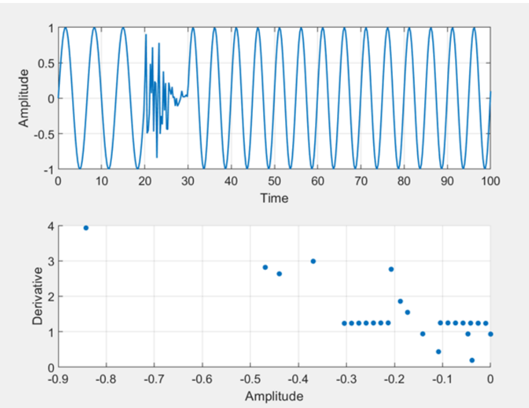

Fig. 16: Time series and Poincaré map for the case of intermittency when V_G is 0.03.

were also varied to observe dynamic behaviour. When inertia was altered between 1.7 kgm^2 and 1.81 kgm^2, intermittent instability emerged. Similarly, machine voltage was varied between 0.05 to 0.03 resulted in an increase in instability due to intermittency.

3 Discussion and Conclusion

The findings of the study highlight the significance of intermittency as a precursor to chaos in the swing equation. Identifying the critical values of the intermittent behaviour can be helpful in detecting instability in power systems. This is particularly relevant in power grids with low inertia that are currently used in the real world.

Comparing the results with existing literature articles, the Lyapunov exponents validation follows similar methodologies [25] to identify chaos. This confirms that phenomena of intermittency can be assessed through exponent fluctuations. Also, the findings are consistent with intermittency seen in fluid dynamics [4] depicting that analysing intermittency is applicable across various nonlinear systems.

This study has some limitations that need to be considered. The swing equation is analysed in an idealised environment without considering external factors such as power grid faults and sudden load changes. Although the inertia and voltage of the machine are studied, other parameters like the damping, and network topology also should be included to study intermittent behaviour.

This paper enhances the understanding of intermittency in power systems by connecting it to the chaotic transitions to the swing equation, hence paving the way for engineers to reduce the adverse effects.

This research analysed intermittency in the swing equation, depicting its role as a precursor to chaos in power systems. Bifurcation diagrams, Lyapunov exponents, heat maps and Poincaré maps were employed to identify intermittency. The results confirm that intermittency can also be observed as the inertia and voltage of the machine are varied, reinforcing the importance of studying small disturbances in the system.

Power grids which use renewable energy integration with low inertia exhibit intermittent behaviour. This study highlights that low-inertia systems show increased opportunities for intermittent instability, depicting the importance and the need for stability control mechanisms. Additionally, the analysis of the voltage of the machine suggests that accurate voltage regulation strategies can reduce chaotic transitions.

The results of this paper provide a foundation for developing early warning signs for the power grids to become unstable. Implementing Lyapunov exponent monitoring tools to detect intermittent behaviour within the system.

Although the study contributes to further analysis within nonlinear systems, it has limitations that pave the way to future research. The swing equation analysis did not consider external factors such as load variations, stochastic fluctuations or grid faults. In the future, studies should focus on power systems in real-time conditions to assess intermittency and other precursors to chaos. It can also be extended to study multi-system power grids to observe resonances, intermittencies and chaos when the interconnected systems are increased.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process.

The authors wrote, reviewed and edited the content as needed and they have not utilised artificial intelligence (AI) tools. The authors take(s) full responsibility for the content of the publication.

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