Reciprocally Supported Elements (RSE) in Space Structure Configurations.

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2nd International Summit on Civil, Structural and Environmental Engineering

ISCSEE2024, March 19, 2024

Florence, Italy

Summary

- Introduction
- Applications
- RSE Transformation and Optimisation
- Case Study
- Structural Modelling and lab output
- Required further research
- Conclusions

Introduction

- Creative engineered architectural forms can be achieved when using multiple circuit arrangements of reciprocally supported elements (RSE).

- Appearance often similar to woven basket assemblies.

- Structural elements of various cross-sections and materials can be used.
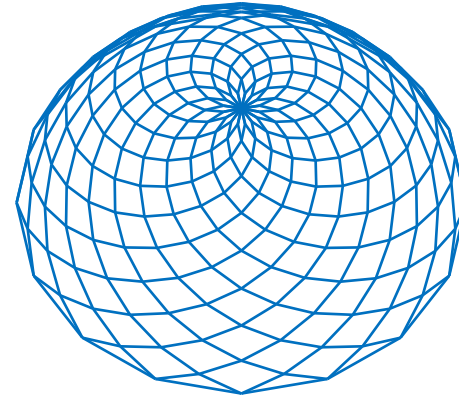
Applications

- Potential applications range from the construction sector to aid work as RSE structures can be mobile and rapidly assembled.

- Economic advantage over the more traditional connection systems where, for example, machined cast ball-joints connectors are employed.
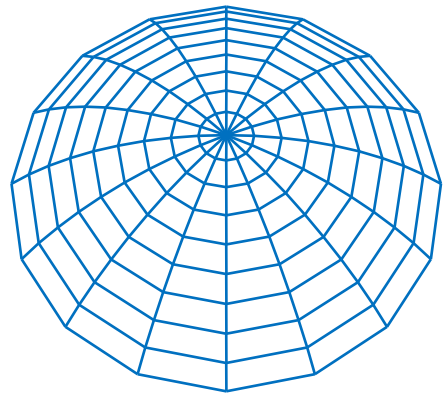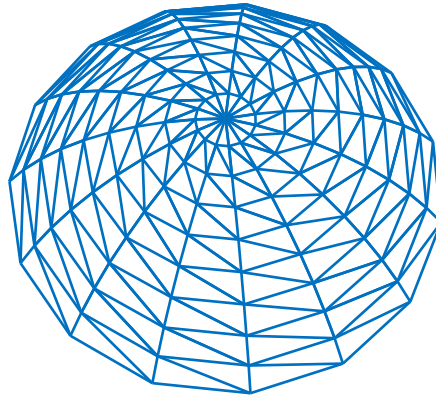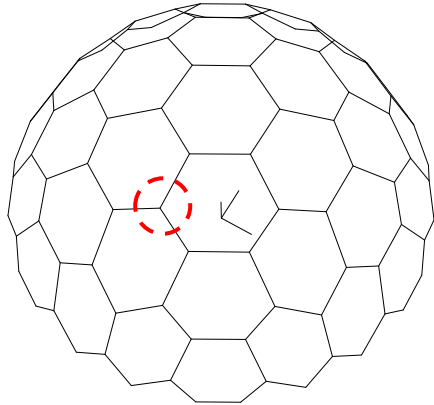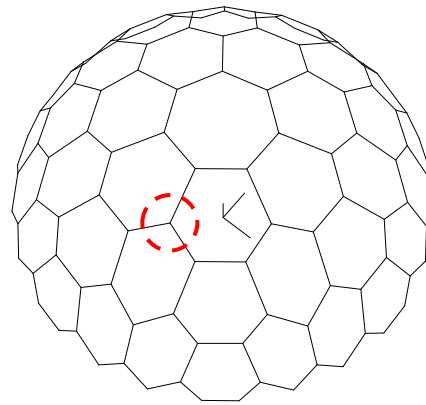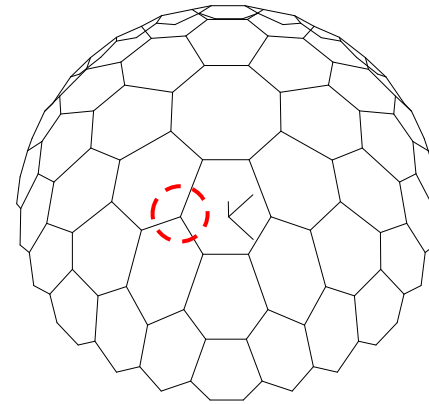
(a)   (b)   (c)

(e)   (f)   (g)

Typical Formian* generated honeycomb domes with frequency, m=3, sweep angle, A=60 degrees and (a) n=6, (b) n=7 and (c) n=8.

*Software developed at the University of Surrey, UK
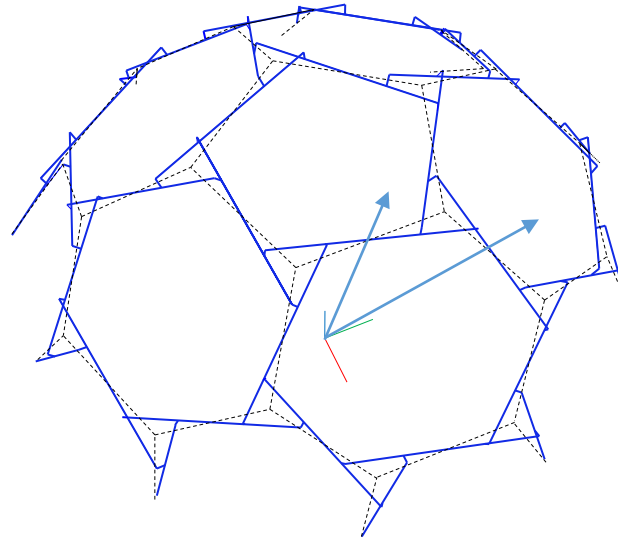
Elementary dome shapes for RSE transformation

- Diamatic domes are another family of lattice domes along with the (a) Ribbed, (b) Schwedler and (c) Lamella type.
- Honeycomb diamatic domes frequently used in practice due to the avoidance of element cluttering near the crown.
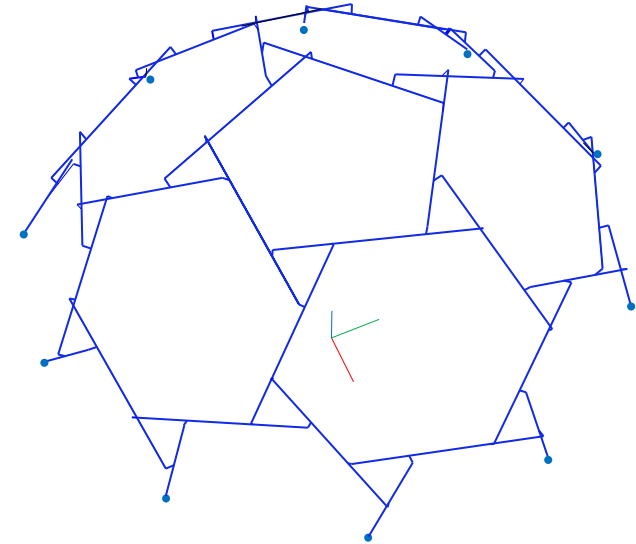- Convenient as RSE configuration processing greatly simplified.

RSE Transformation – Formian and Rhinoceros
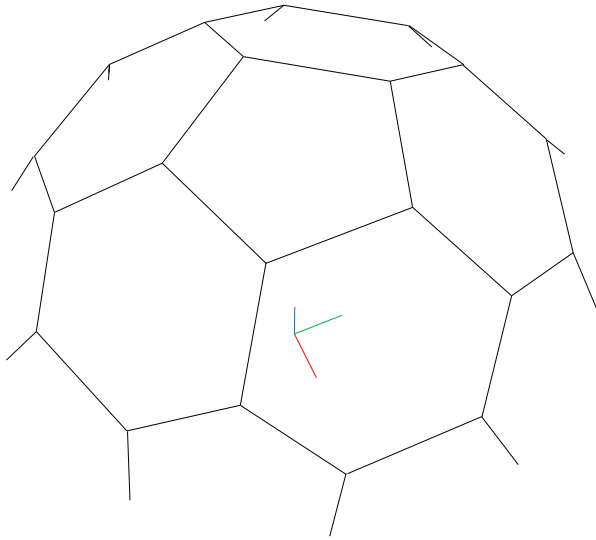
(a)   (b)   (c)

Perspective views of (a) elementary dome, (b) rotation method transformation and (c) RSE honeycomb dome using initial rotation angle of 15^0.

Case Study – RSE Honeycomb Dome

- Study aim was to compare predicted behaviour with monitored behaviour in the laboratory.

- Part 1 of the study was structural modelling.

- Part 2 involved manufacture, construction, loading and monitoring in the laboratory.

- Experimental output would allow modelling calibration.

Transformation Optimisation

Dome construction

(i) 48.3mm dia. cylindrical Circular Hollow Section (CHS) tubes.

(ii) 12mm diameter bolts in 13.0 mm diameter clearance (oversized) holes.

(iii) Saddleback washers used with a minimum thickness of 0.85 mm for accurate seating and location.

(iv) Modified rotation method was used to achieve a 50mm (+) 2.5 mm or (-) 0.5 mm target eccentricity.

(v) RSE dome span and rise was determined as 3066 mm and 894 mm respectively.
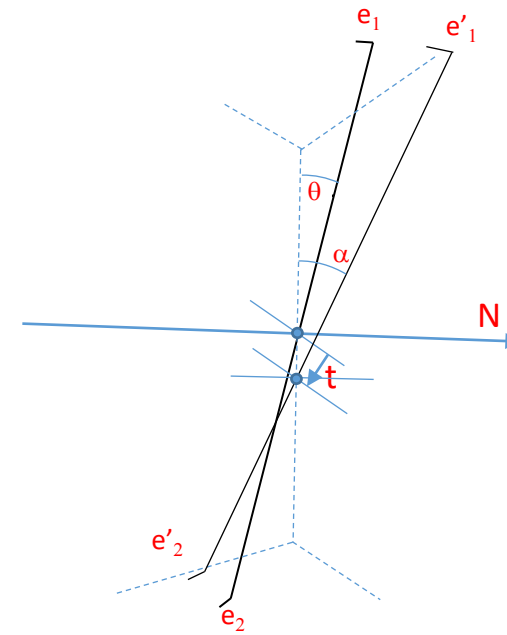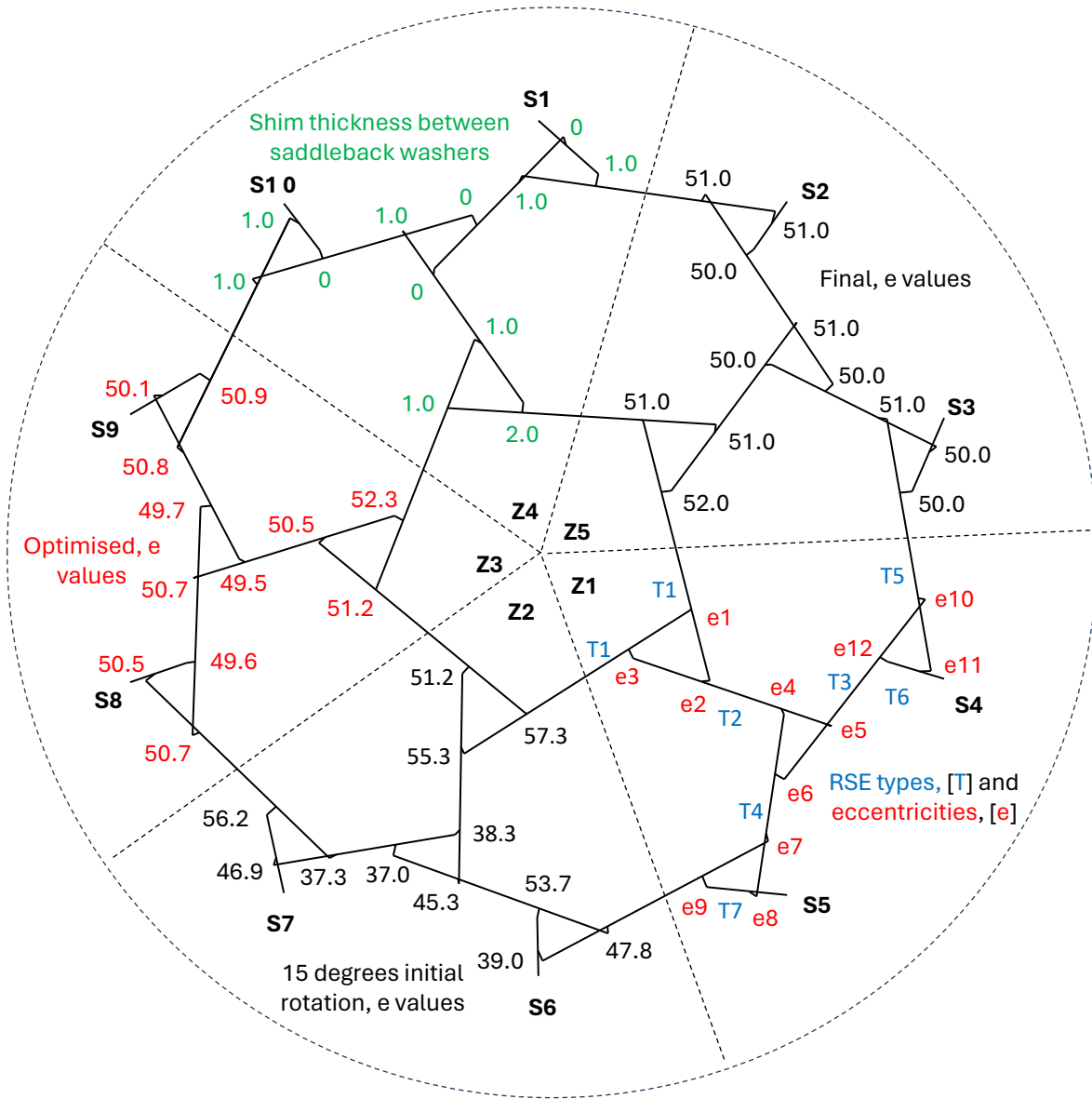
Eccentricity Optimisation

RSE Honeycomb dome plan.
Boundary supports, [S1 - S10].
Elements types, [T1 - T7].
Bolted connections eccentricity, [e1 - e12].

Note: all sector zones [Z1 - Z5] identical with symmetry indicated by dotted lines.
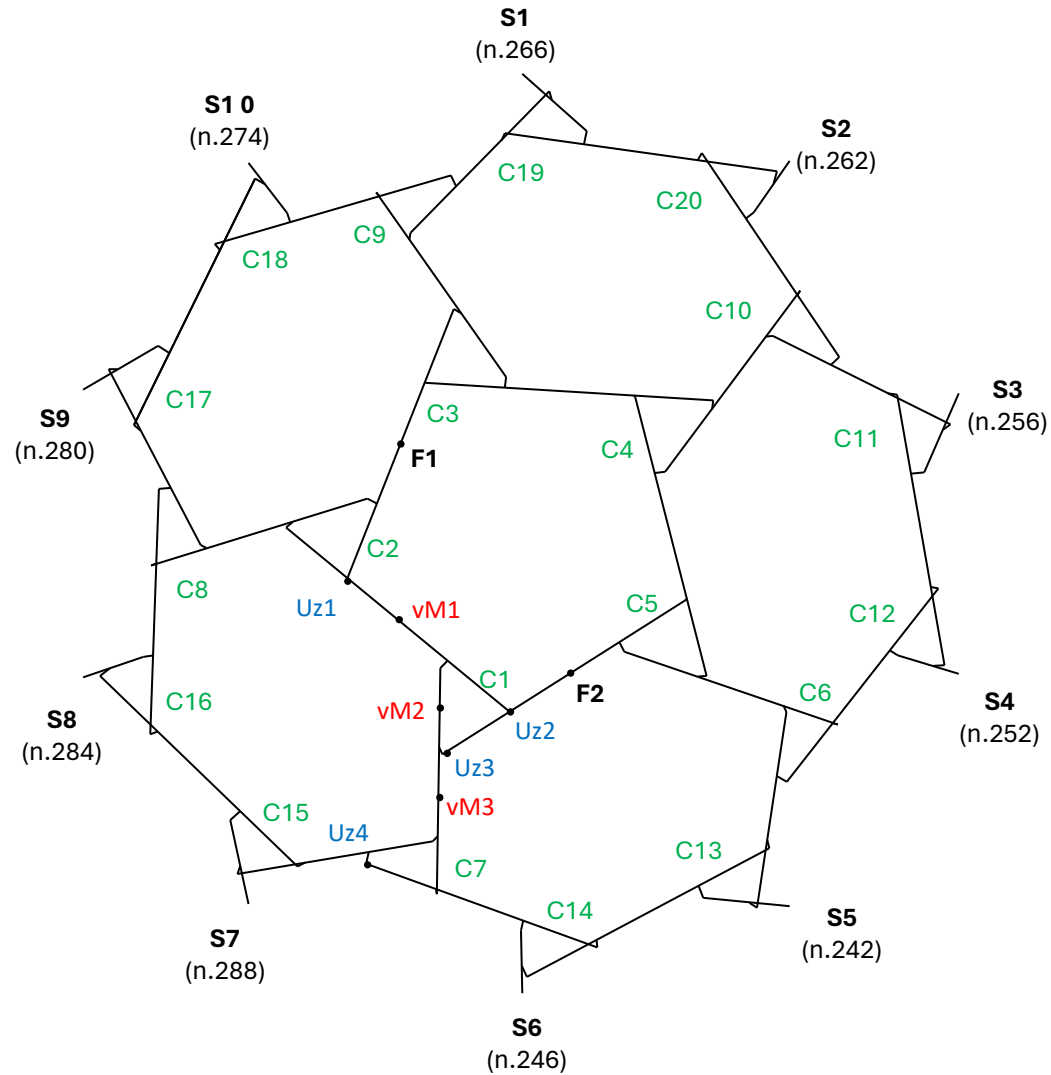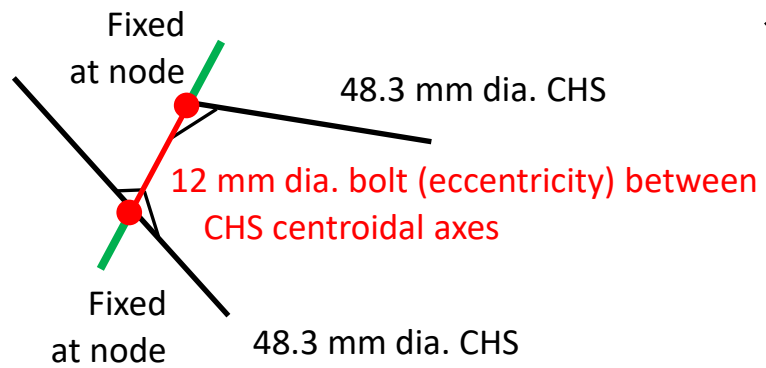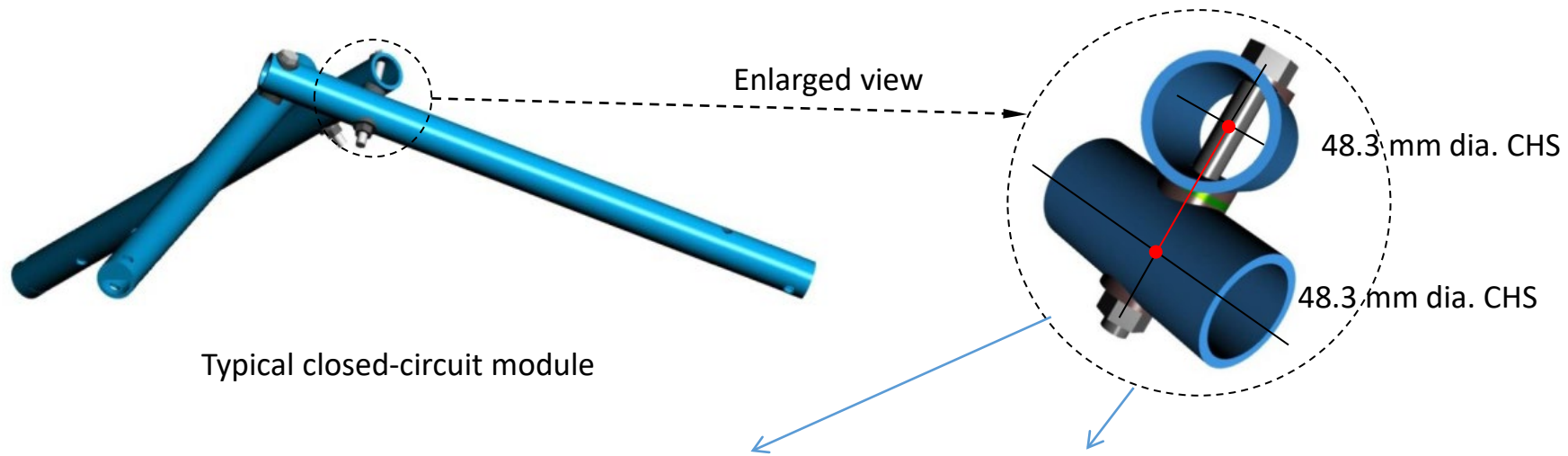
GSA analysis boundary supports node numbering.

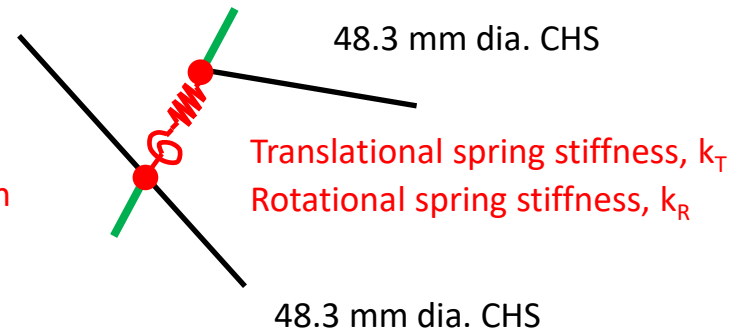Loading positions indicated by [F1 - F2].

Displacement monitoring, [Uz1 - Uz4].

GSA analysis and lab von Mises stresses monitoring locations, [vM1 - vM3].
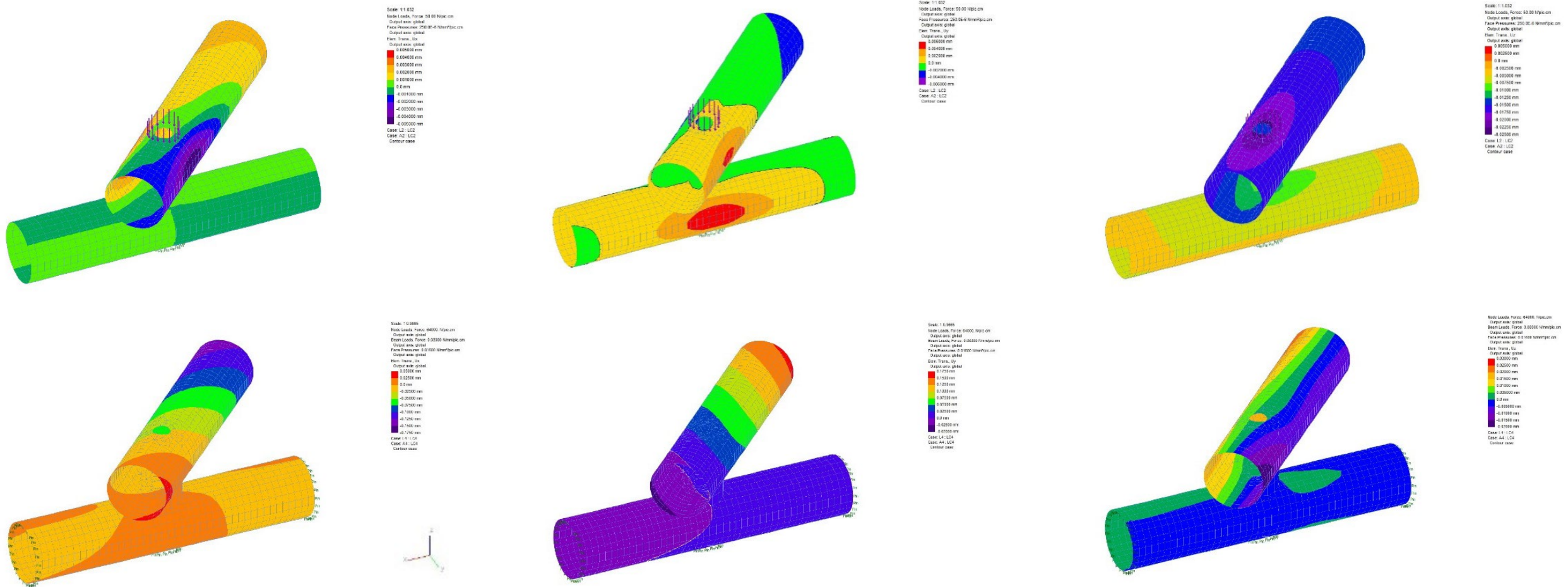
Module circuits numbered [C1 - C20].

Enlarged view

Typical closed-circuit module

48.3 mm dia. CHS

48.3 mm dia. CHS

Fixed at node

48.3 mm dia. CHS

12 mm dia. bolt (eccentricity) between CHS centroidal axes

Fixed at node

48.3 mm dia. CHS

48.3 mm dia. CHS

Translational spring stiffness, k_T
Rotational spring stiffness, k_R

48.3 mm dia. CHS

Model 1.

Model 2.

CHS element bolted connections

FEA connection models used to determine translational, k_T and rotational, k_R spring stiffnesses.

Spring	Type	x/xx		y/yy		z/zz	
		Linear/ curve ref.	Stiffness (kN/m) (kNm/rad.)	Linear/ curve ref.	Stiffness (kN/m) (kNm/rad.)	Linear/ curve ref.	Stiffness (kN/m) (kNm/rad.)
Property 1	Translational	Linear	14364	Linear	10231	Linear	15184
Property 2	Rotational	Linear	29.6	Linear	26.9	Linear	266.7

Model 2

Structural Modelling

Boundary supports

- Objective to model the experimental support conditions.

    (i) Minor geometric self-adjustments would take place within the dome structure when initial loading commenced.

- A range of 8 No possible support leg conditions considered including:

    (i) May be free to move laterally as they would not be mechanically fixed in position,

    (ii) Vertical z-direction restraint + horizontal axial stiffness, kx and ky applied

Connections

    (i) Model 1 – All fixed both ends

    (ii) Model 2 – All with kT and kR spring stiffness.

Model		Boundary	Connection
U	vM	Boundary Support Legs (S1 to S10)	Connections (60)
1	1	All 10 nodes pinned	Model 1: All fixed
2	2	2 nodes pinned (S10 n.274, S1 n.262). 8 nodes horizontal rollers restrained vert. z-dir.	Model 1:
2a	3	3 pinned (S10 n.274, S1 n.262, S6 n.246). 7 nodes horizontal rollers restrained vert. z-dir.	Model 1:
3	4	10 nodes horizontal spring stiffnesses, kx & ky restrained vert. z-dir.	Model 1:
4	5	All 10 nodes pinned	Model 2: All with Translational and Rotational spring stiffnesses, k_T & K_R
2	6	2 nodes pinned (S10 n.274, S1 n.262). 8 nodes horizontal rollers restrained vert. z-dir.	Model 2:
5a	7	3 pinned (S10 n.274, S1 n.262, S6 n.246). 7 with horizontal rollers restrained vert. z-dir.	Model 2:
6	8	10 nodes horizontal spring stiffnesses, kx & ky restrained vert. z-dir.	Model 2:

Analysis Models

Linear Elastic Analysis

Arup Oasys GSA 8.7 (General Structural Analysis) software with 3-dimensional and finite element capabilities used.

- Two property types defined
    - 48.3 mm diameter 4.0 mm thick grade S355 CHSs
    - 12 mm diameter grade M8.8 bolts used for connecting the CHSs together in closed triangulated circuits.

- Two property types defined the translational and rotational stiffnesses.

- Applied load range of 1 kN to 8 kN

- Varying boundary supports and connections models considered.

Output.

- Displacements, Ux, Uy and Uz

- Von Mises stresses,

$$\sigma_{VM} = (\sigma_{xx}^2 + 3\tau_{xy}^2 + 3\tau_{xz}^2)^{0.5}$$

$$\leq \sigma_y \text{ (yield strength of material) monitored}$$
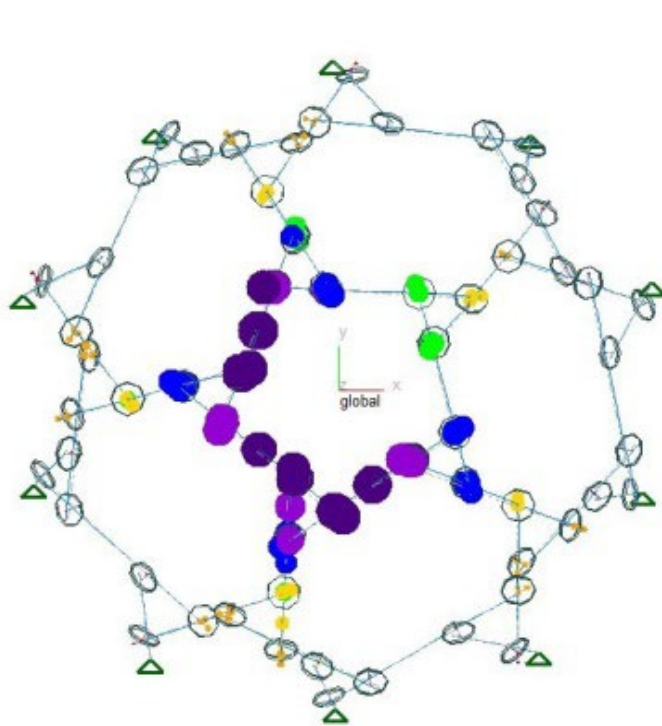
Element list: G1 G2 G3 G4 G5 G6 G7 G9
Scale: 1:27.0
Deformation magnification: 25.00
Translation, Uz: 10.00 mm/pic.cm
Output axis: global

0.2670 mm
-0.7856 mm
-1.838 mm
-2.891 mm
-3.943 mm
-4.996 mm
-6.049 mm
-7.101 mm

Case: L3 : LC3
Case: A3 : LC3
Contour case
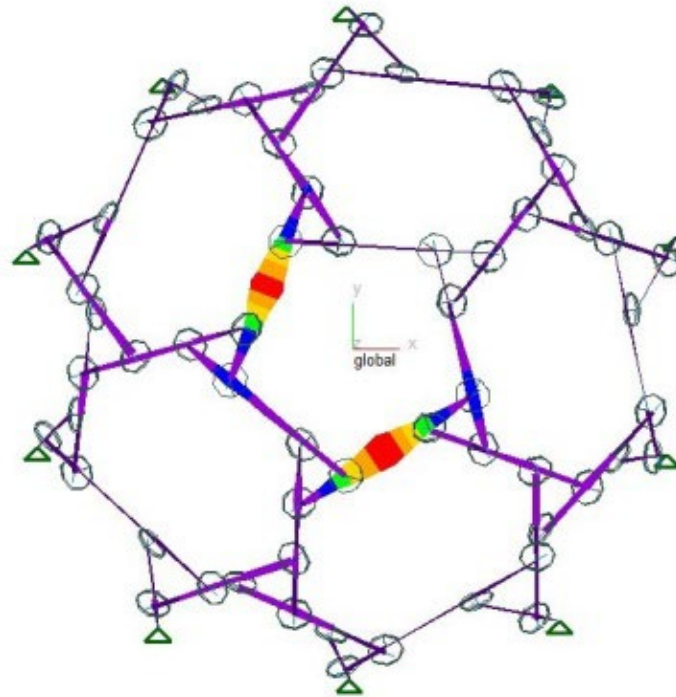
Element list: G1 G2 G3 G4 G5 G6 G7 G9
Scale: 1:27.0
Beam Stress, Von Mises: 250.0 N/mm²/pic.

151.0 N/mm²
129.5 N/mm²
107.9 N/mm²
86.30 N/mm²
64.73 N/mm²
43.15 N/mm²
21.58 N/mm²
7.985E-6 N/mm²

Case: L3 : LC3
Case: A3 : LC3
Contour case

(Model 5) All supports pinned. All connections with spring stiffnesses.
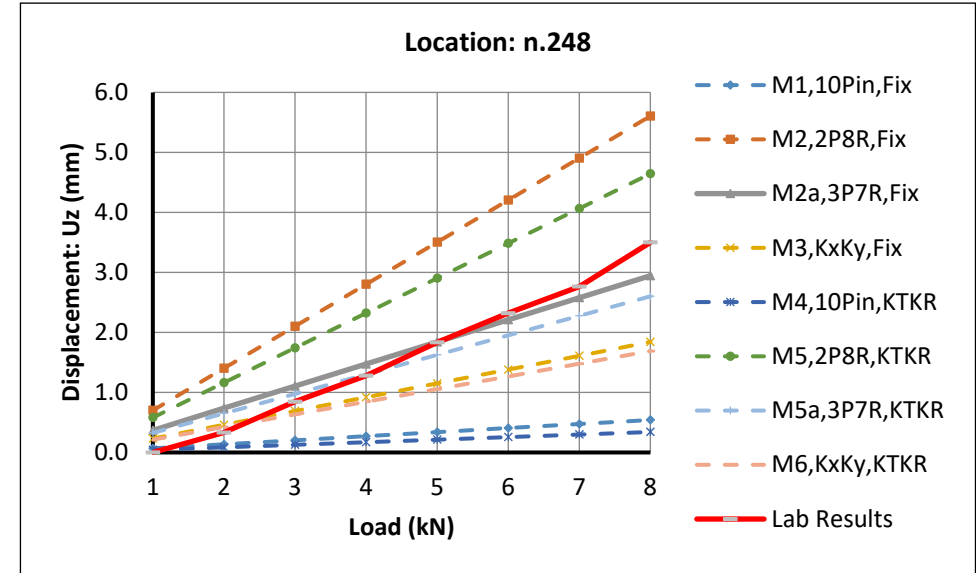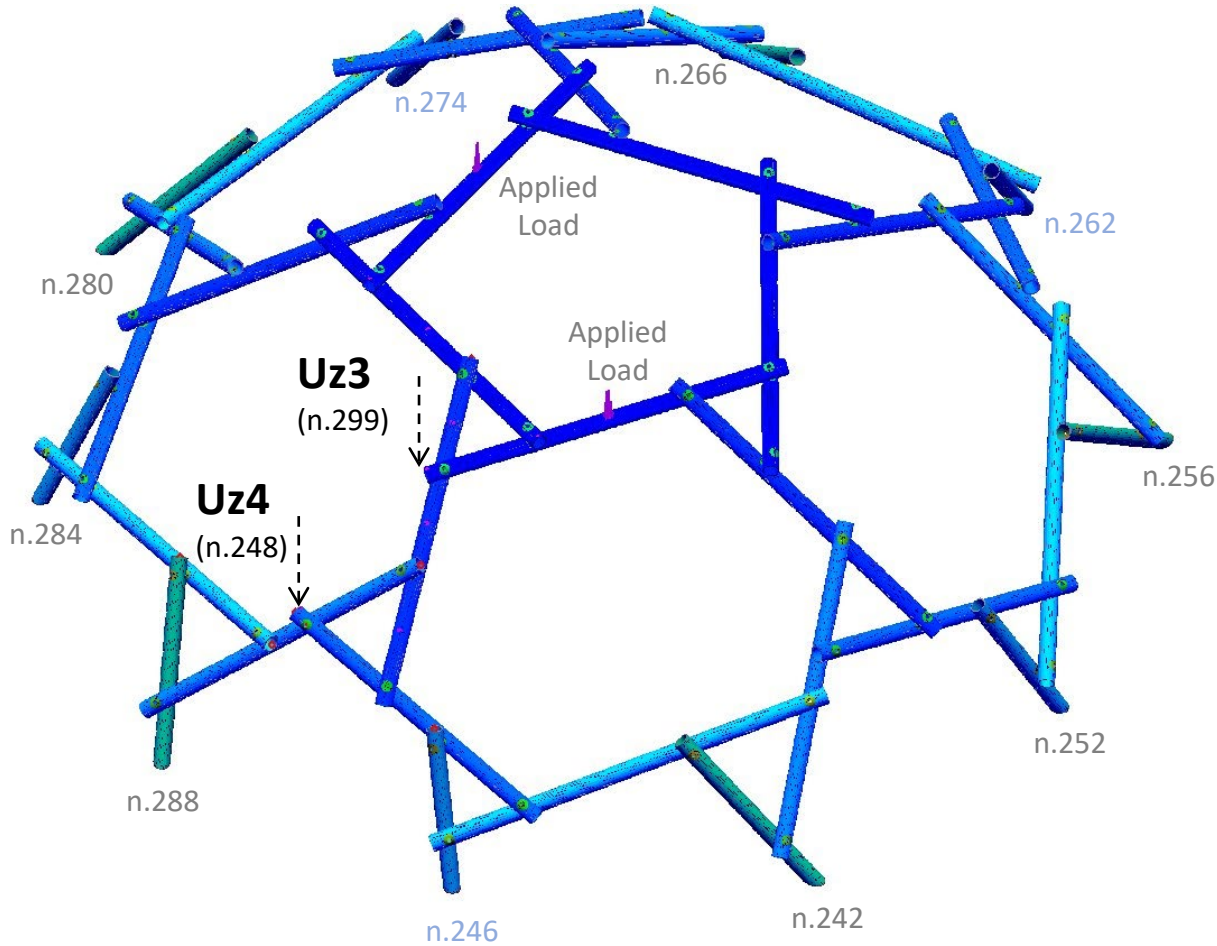
RSE honeycomb dome being constructed.
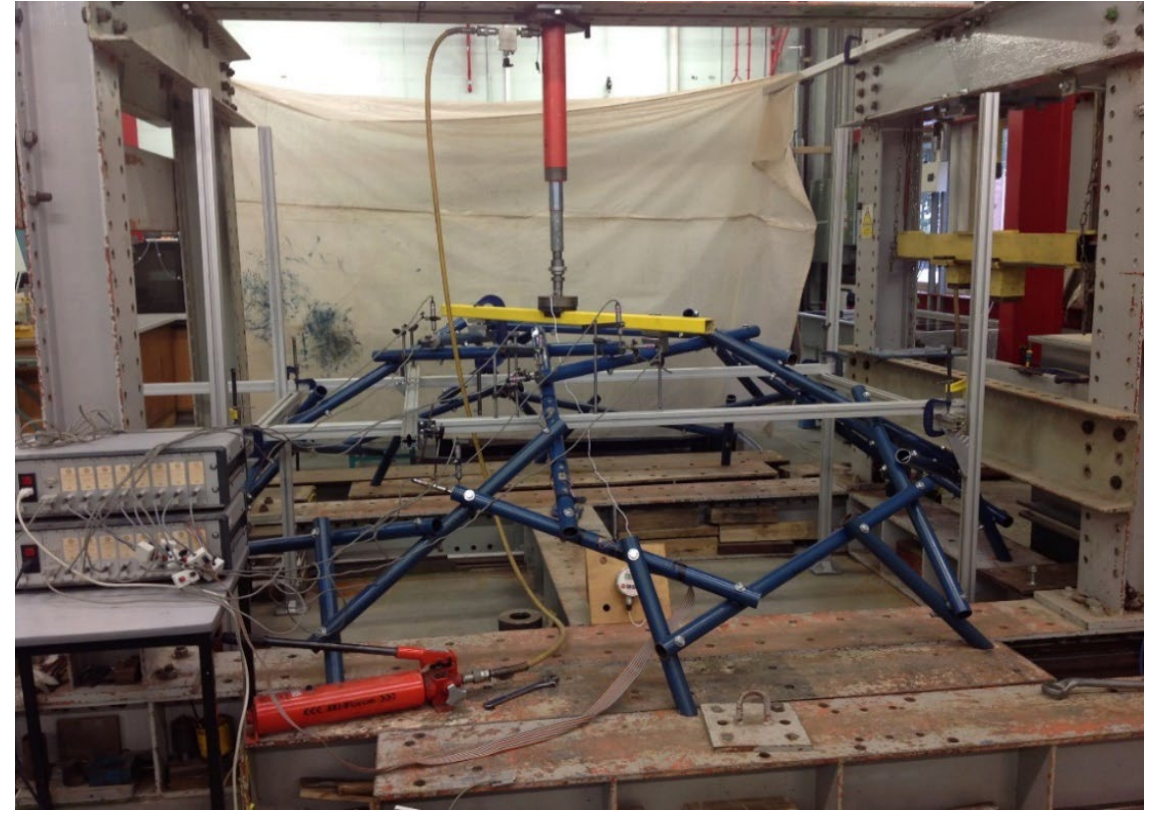
RSE honeycomb dome in test setup.

- Rosette stacked strain gauge matrix orientation on upper tube surfaces at monitoring locations.

- Hydraulic ram loaded spreader beam and CHS tube bearing assembly with LVDT set up for displacement monitoring.
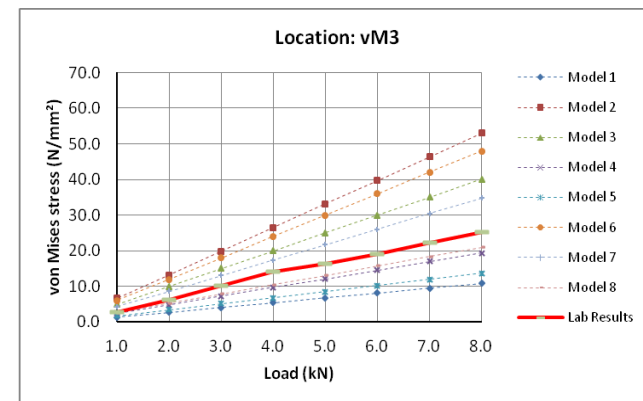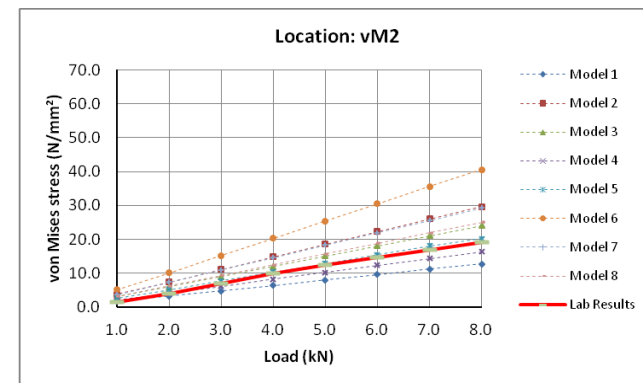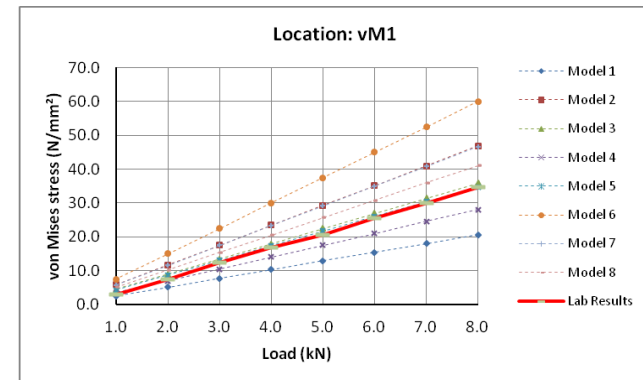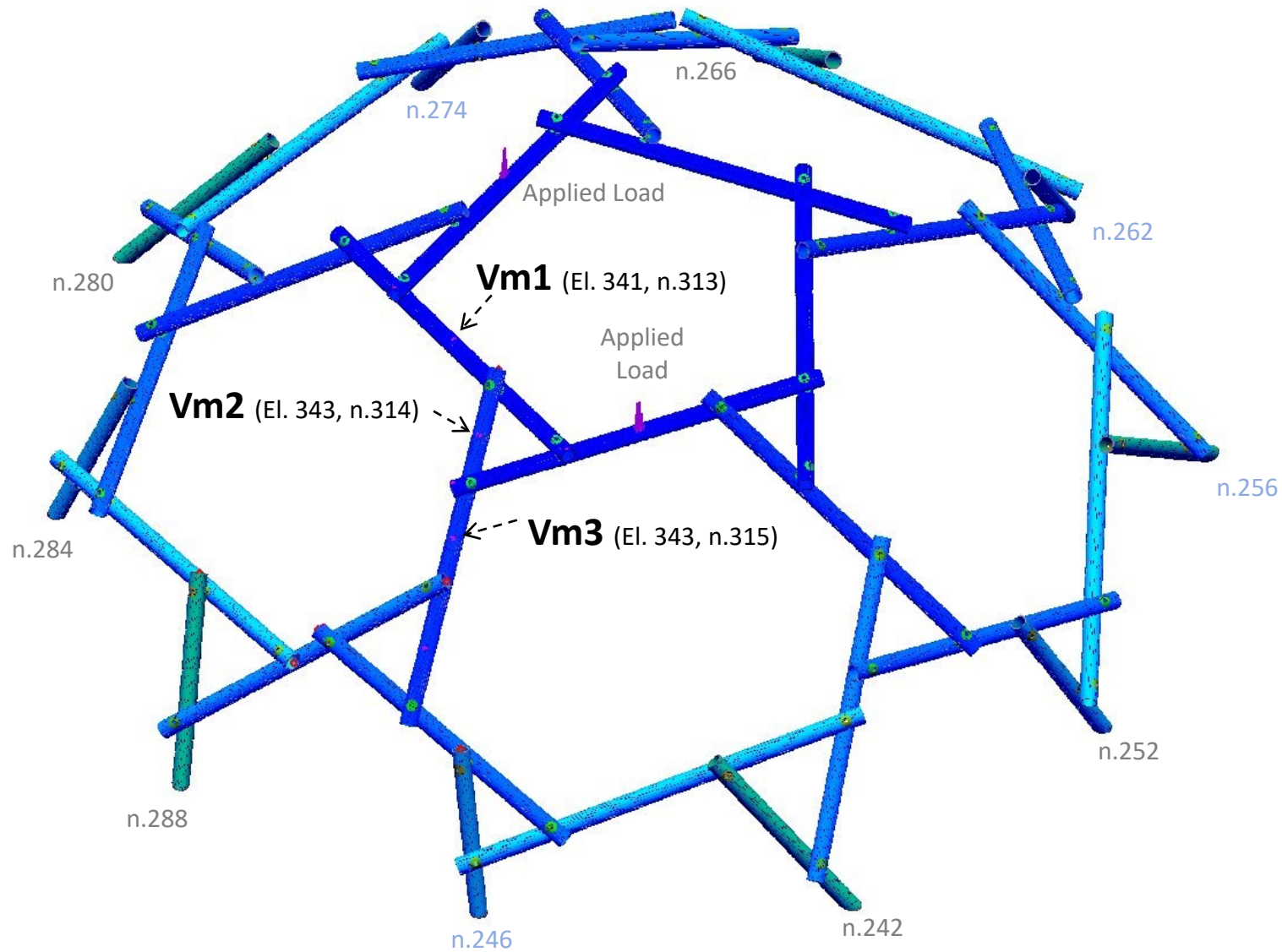
Displacement Monitoring
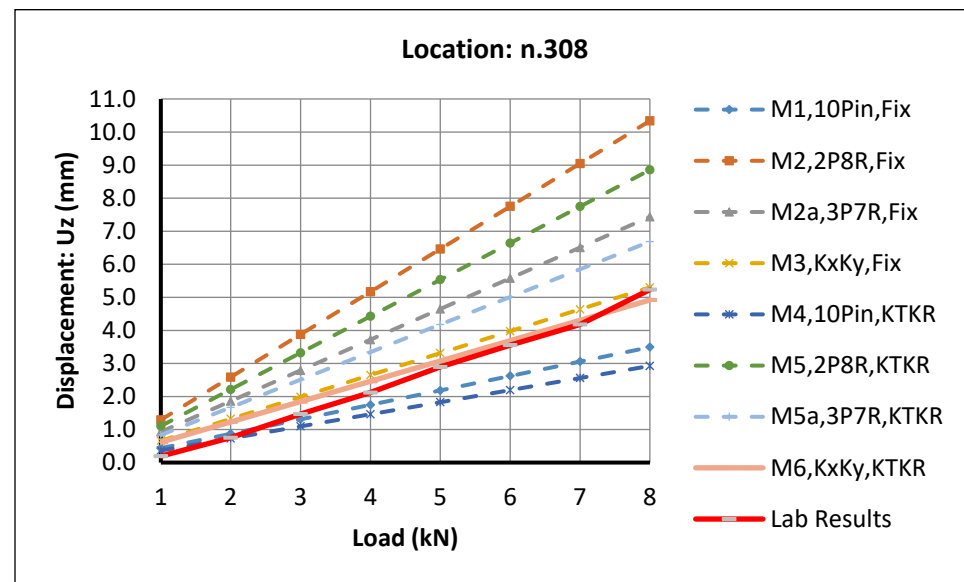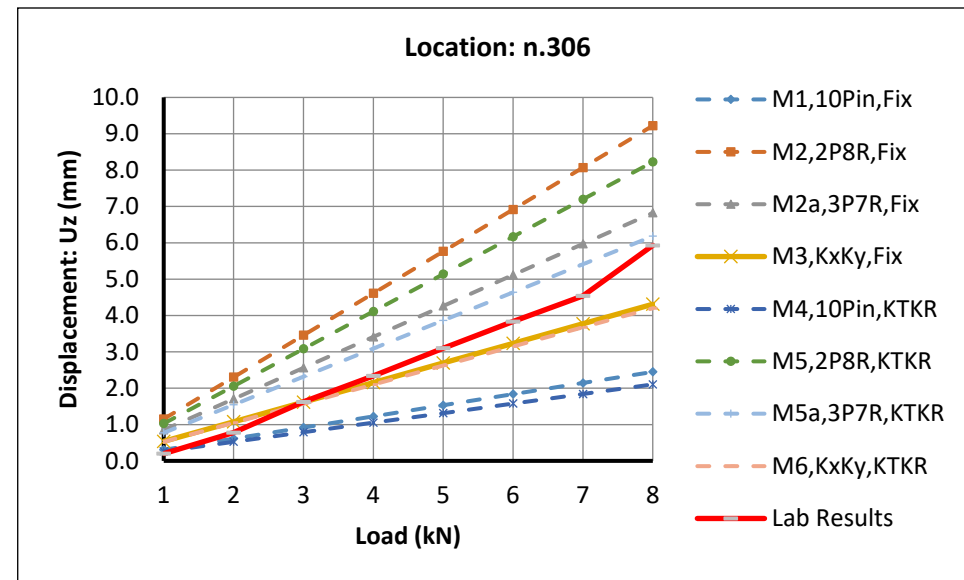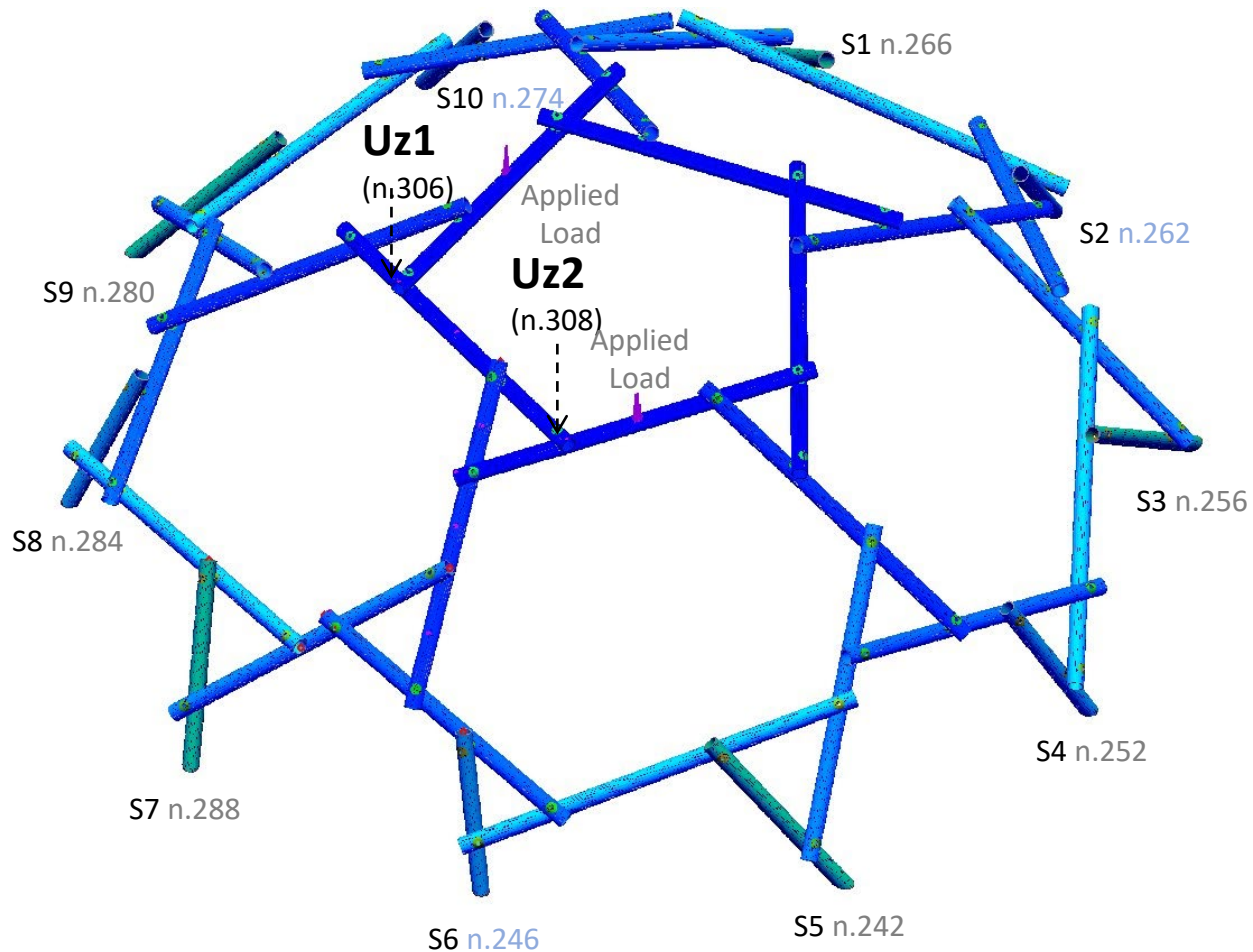
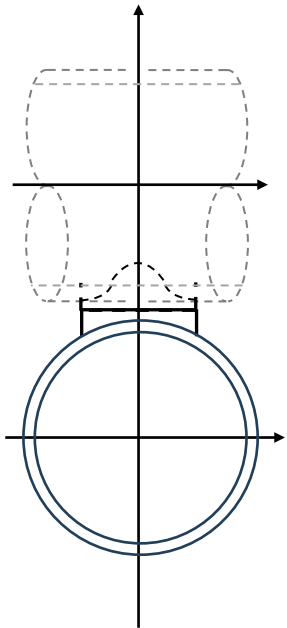# Displacement Monitoring
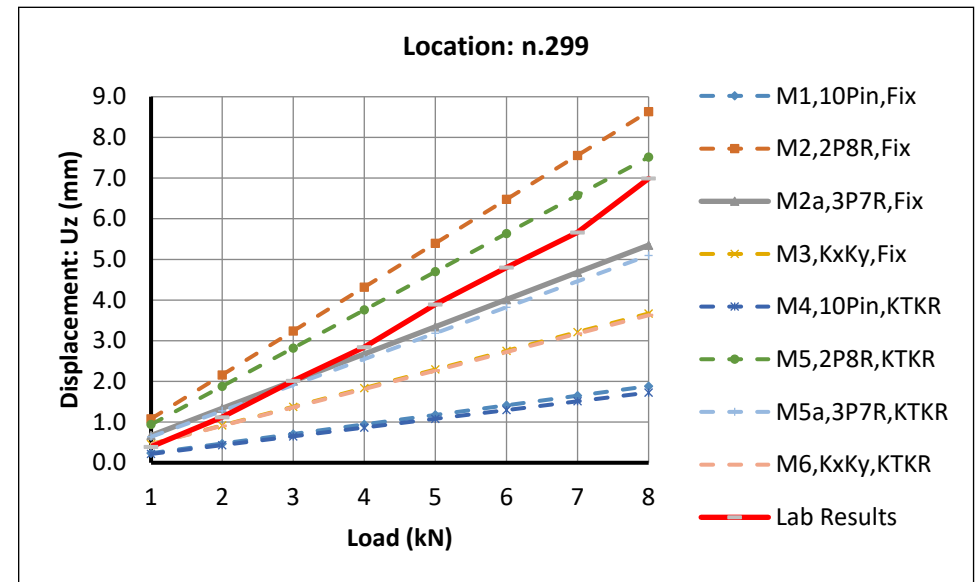
Von Mises monitoring locations

Range of analysis models represent behaviour - further Research required

Important factors in numerical modelling of RSE space structures

(i) The stiffness of the connection associated with CHSs.

(ii) CHSs incident angles.

(iii) Load distribution in connection elements

(iv) Effective loaded CHS width

(v) Boundary support stiffness

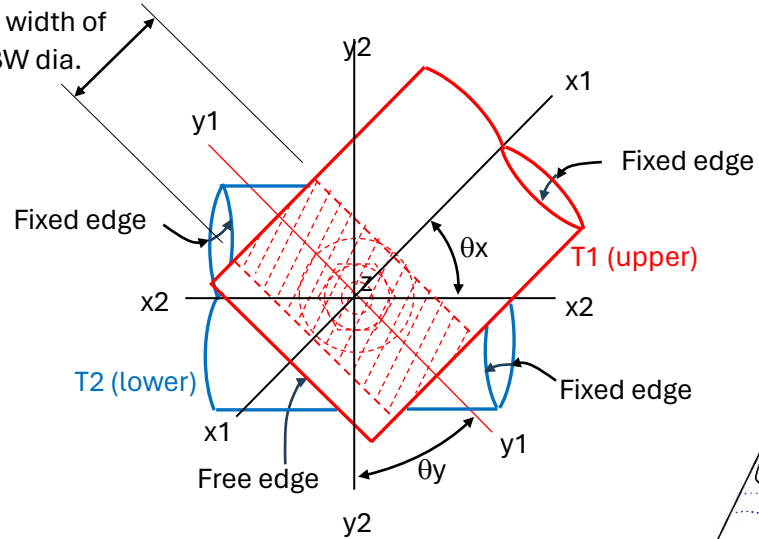Study widened to determine stiffness of

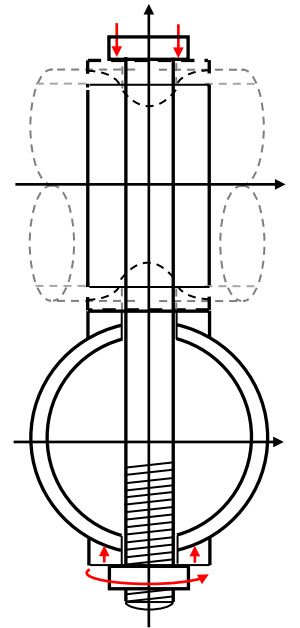(i) RSE CHS welded connections

(ii) RSE CHS bolted connections

T1 and T2 effective width of loaded CHS = SBW dia.

Fixed edge

Fixed edge

θx

T1 (upper)

x2 — x2

T2 (lower)

Fixed edge

x1

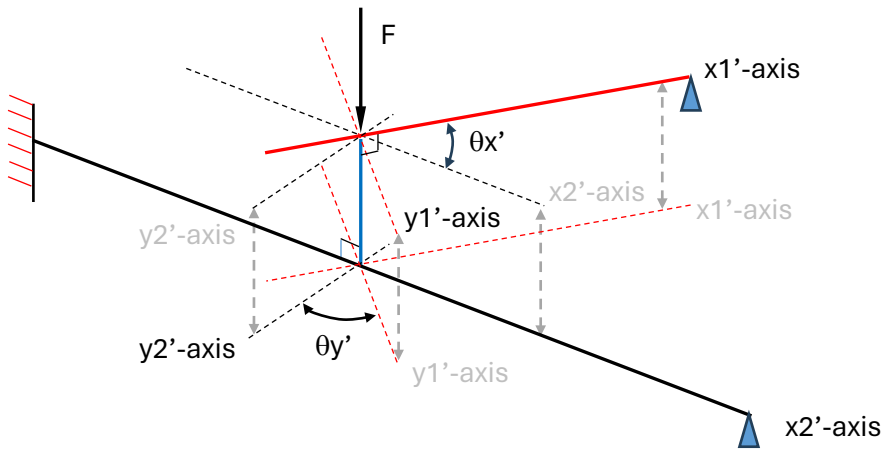Free edge
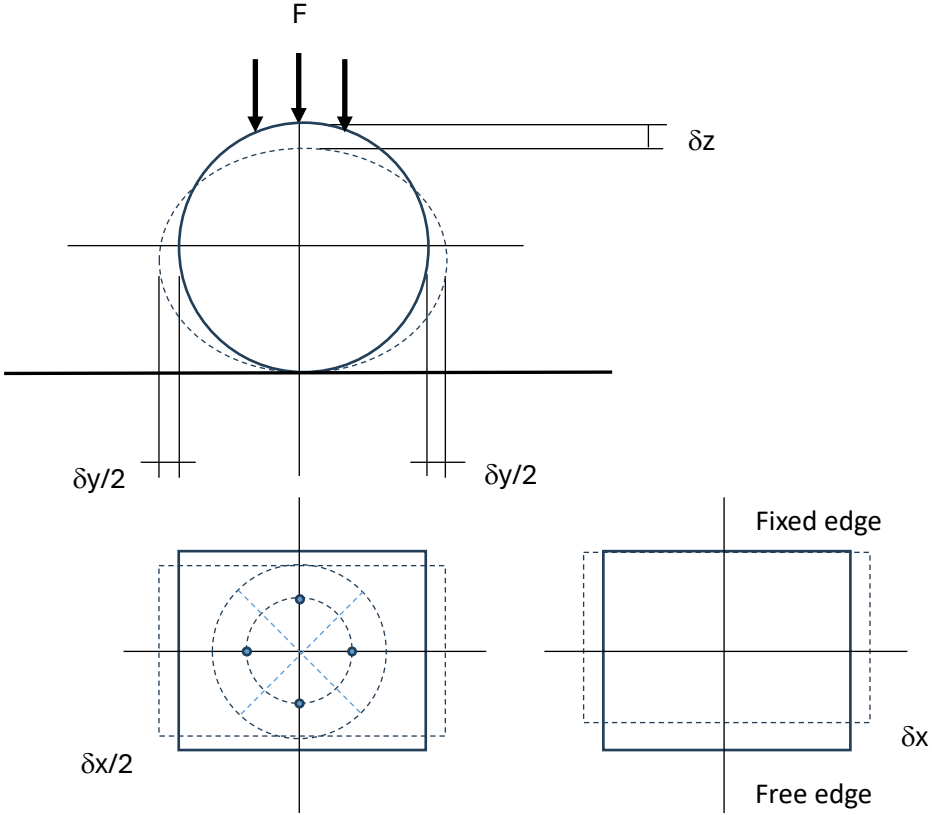
θy

y1

Definitions and assumptions CHSs at θ degrees to each other
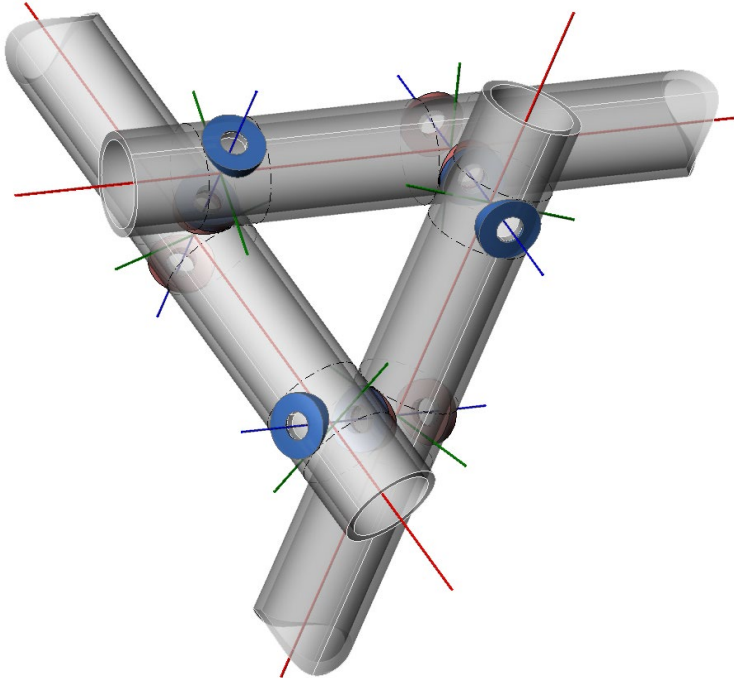
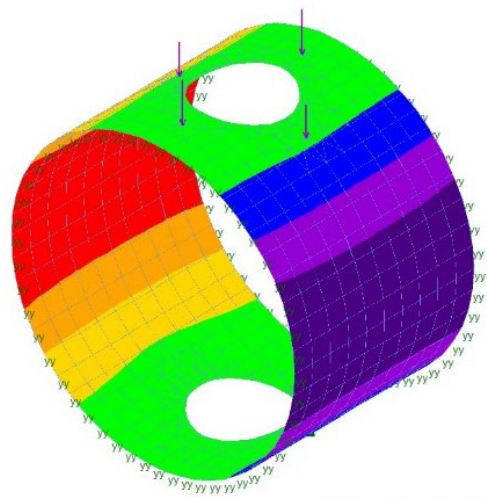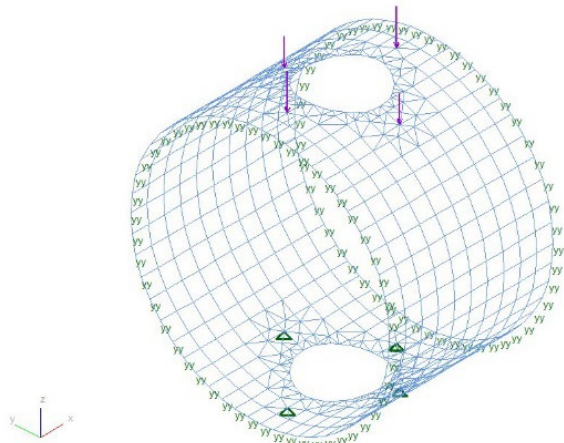Spring stiffness: Bolt in tension, CHS in compression

Spring stiffness: CHS and weld

Welded RSE connection with SBW used for locating CHSs.

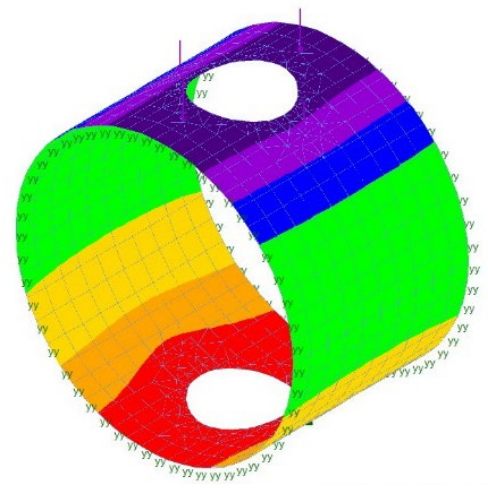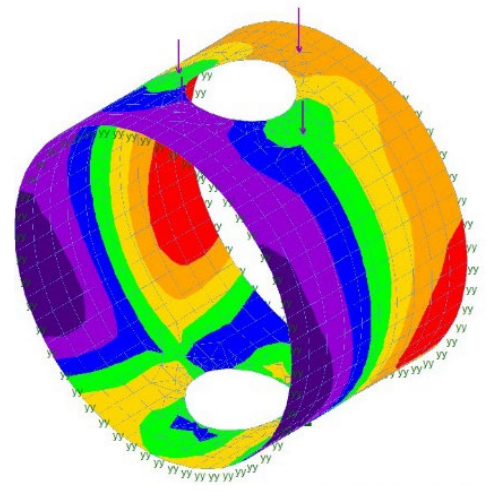Bolted RSE connection with SBW and clearance (oversized) holes.

Translational stiffness

CHS local axes and angles of incidence

Translational stiffness – effective width FEA models

Rotational stiffness

Local x'-axis

Local y'-axis

Local z'-axis

Eccentricity, e midpoint
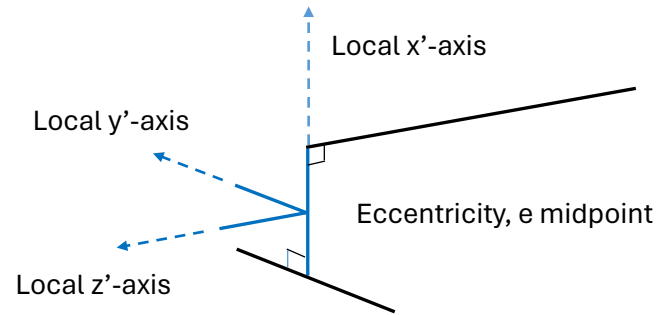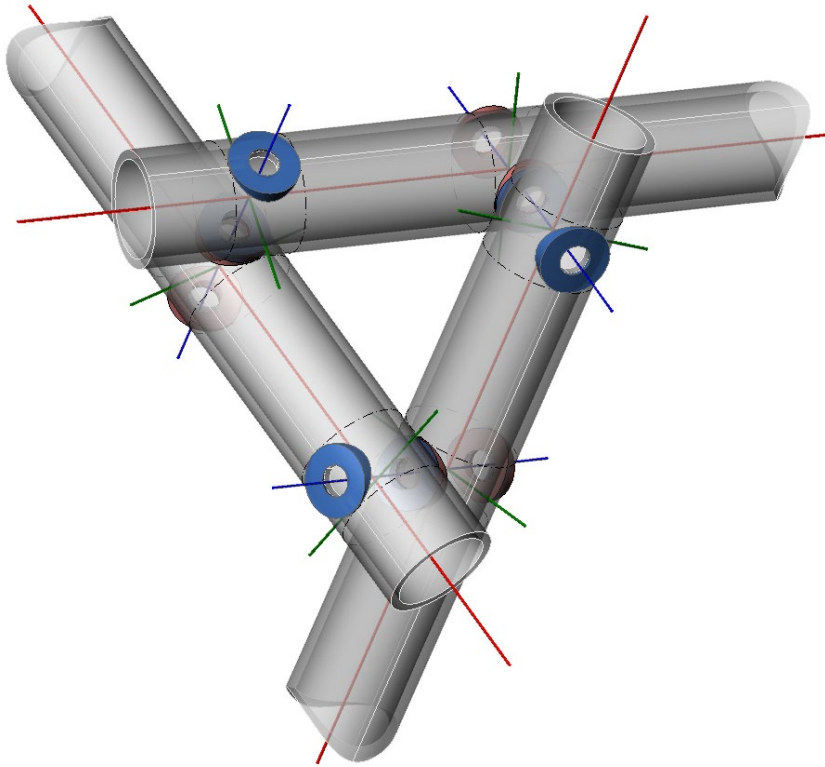
CHS connection rotation stiffness

Fv @ y'-axis, -- Mz'

Fh @ y'-axis, -- Mx'

Fv @ z'-axis, -- My'

x'-axis

y'-axis

Fv

z'-axis

x'-axis

y'-axis

Fh

z'-axis

x'-axis

y'-axis

Fv

z'-axis

Fv

La

θ

δ_{lvdt}
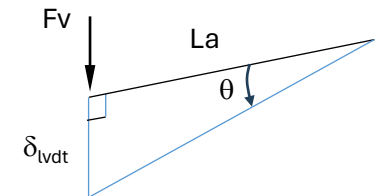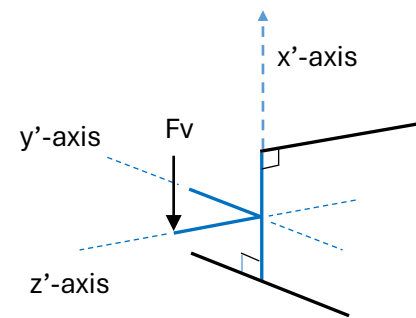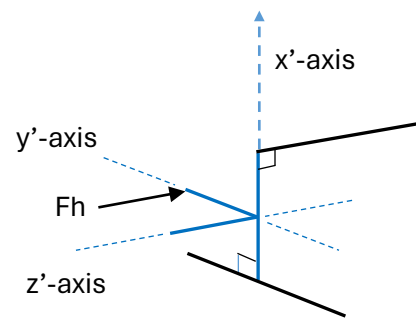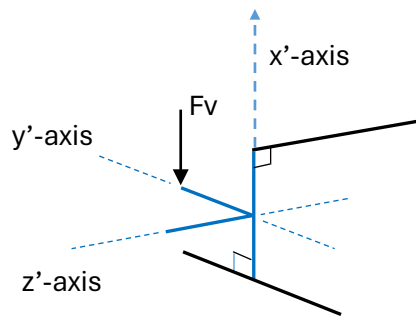
$My' = (Fv \times La)$ (kNm)

$\theta = \tan^{-1}(\delta_{lvdt}/La \times \pi/180)$ (radians)

$k_R = M/\theta$

Linear springs
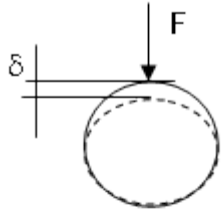
$kT = F/\delta$

(kN)/(m)
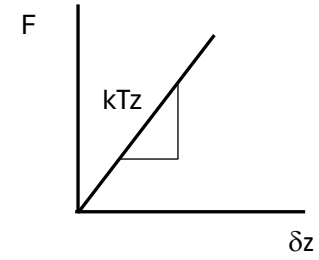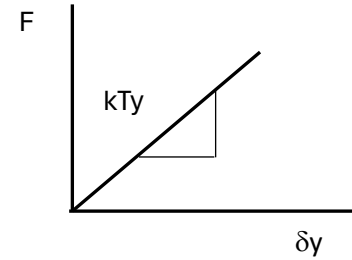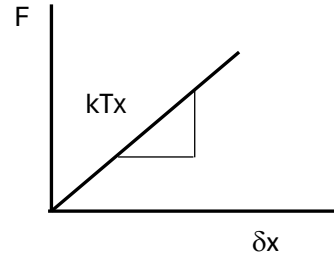
Elastic Translational stiffness, kTx, kTy, kTz
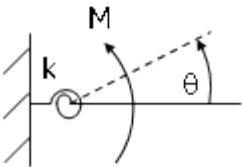
$kR = M/\theta$

(kNm)/(Radians)
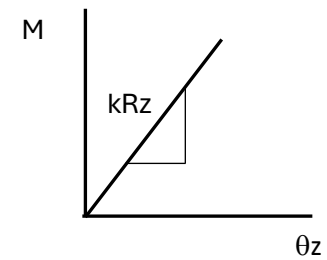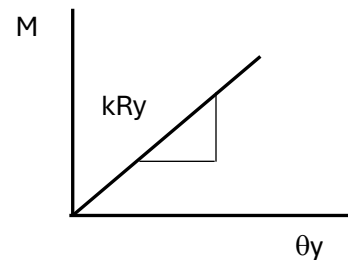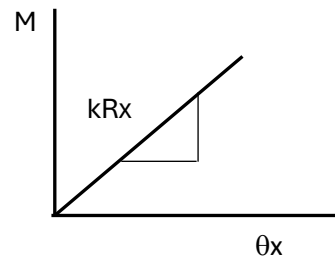
Elastic Rotational stiffness, kRx, kRy, kRz

Torsional springs
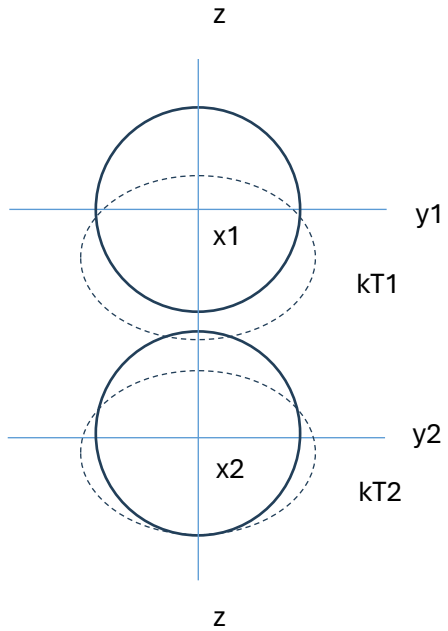
CHS connection translational stiffness

$kTz = Fz'/\delta z'$

$kTx = Fx'/\delta x'$

$kTy = Fy'/\delta y'$
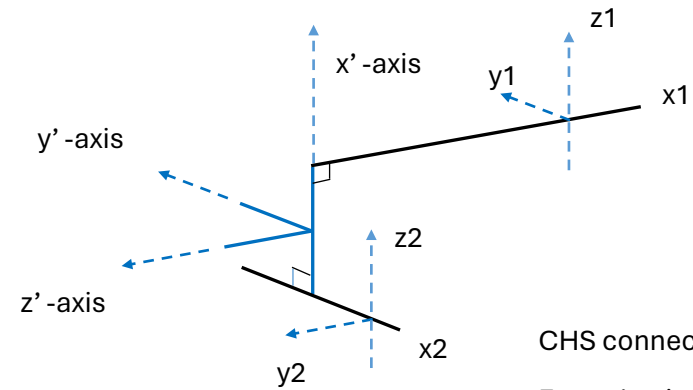
CHS connection rotation stiffness

Fv @ y'-axis, -- Mz' $\qquad$ $kRz = Mz'/\theta z'$

Fh @ y'-axis, -- Mx' $\qquad$ $kRx = Mx'/\theta x'$

Fv @ z'-axis, -- My' $\qquad$ $kRy = My'/\theta y'$

RSE welded/bolted connection elastic stiffness					
Translational stiffness			Rotational stiffness		
x-axis	y-axis	z-axis	x-axis	y-axis	z-axis
springs in parallel $Kx = k1 + k2$	springs in parallel $Ky = k1 + k2$	springs in series $1/Kz = 1/k1+1/k2$ $Kz = K1K2/k1 + k2$	kRx	kRy	kRz

Conclusions

Initial Study

- Finite element connection models used to determine spring stiffness used for global analysis.

- Unrealistic high values of stress apparent at connections with assumed full fixity.

- Spring elements developed more representative stresses.

Further investigation identified

- Connection stiffness graphs for varying CHS incident angles

- Effective width of CHS

- Load distribution mechanics

- Welded and bolted connection stiffness differences

References

•Saidani, M. and **Rizzuto, J.P.** (2000) Proc. Int. Colloquium on Structural Morphology, Engineering Design Assisted by Physical Models, Delft, The Netherlands, The International Association for Shell and Spatial Structures. pp. 422-426.

•**Rizzuto, J.P.**, Saidani, M. and Chilton, J. C. (2000) The Self-Supporting Multi-Reciprocal Grid (MRG) System Using Notched Elements Journal of The International Association for Shell and Spatial Structures **41** (2) (133) August, pp. 125-131 ISSN: 0304 3622.

•**Rizzuto, J.P.**, Saidani, M. and Chilton, J. C. (2001) Proc. Int. Symp. on Theory, Design and Realization of Shell and Spatial Structures, Extended Abstracts. Joints and Orientation of Module Elements in Multi-Reciprocal Grid (MRG) Systems, Nagoya, Japan, The International Association for Shell and Spatial Structures. pp. 308-309.

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Thank you for listening

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