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Reciprocally Supported Elements (RSE) Space Structure Configurations

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Reciprocally Supported Elements (RSE) in Space Structure Configurations.

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2nd International Summit on Civil, Structural and
Environmental Engineering

ISCSEE2024, March 19, 2024

Florence, Italy

Summary

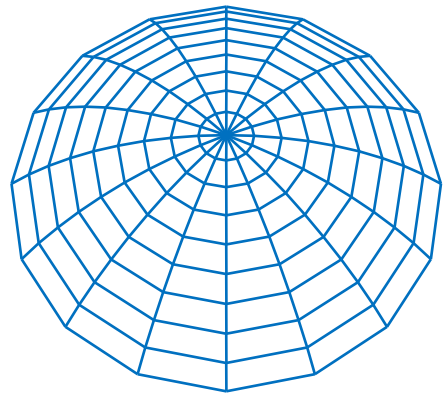
- Introduction
- Applications
- RSE Transformation and Optimisation
- Case Study
- Structural Modelling and lab output
- Required further research
- Conclusions

Introduction

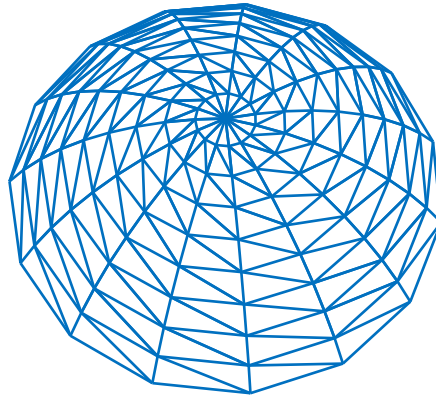
- Creative engineered architectural forms can be achieved when using multiple circuit arrangements of reciprocally supported elements (RSE).
- Appearance often similar to woven basket assemblies.
- Structural elements of various cross-sections and materials can be used.

Applications

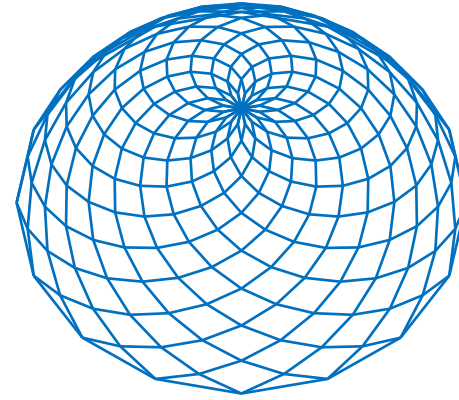
- Potential applications range from the construction sector to aid work as RSE structures can be mobile and rapidly assembled.
- Economic advantage over the more traditional connection systems where, for example, machined cast ball-joints connectors are employed.



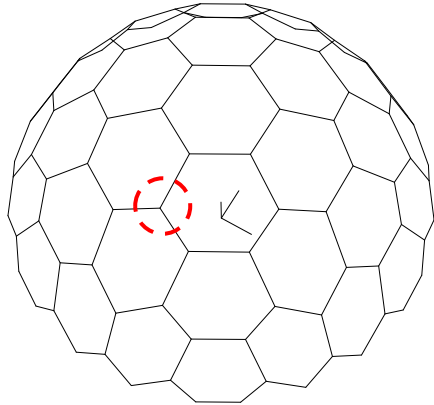
(a)



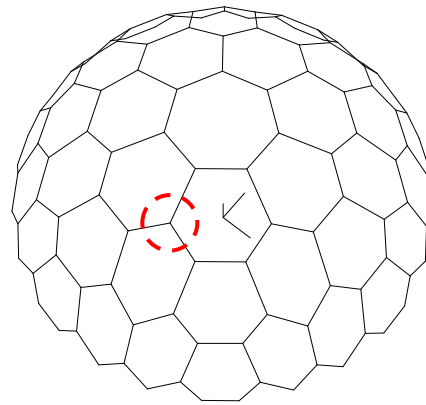
(b)



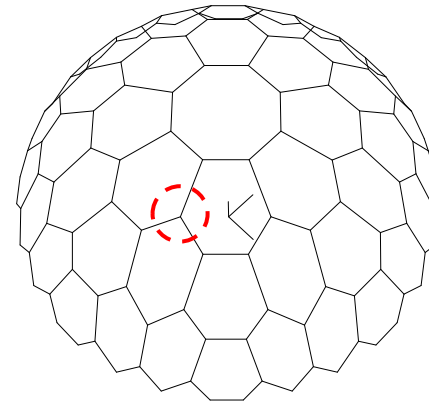
(c)



(e)



(f)



(g)

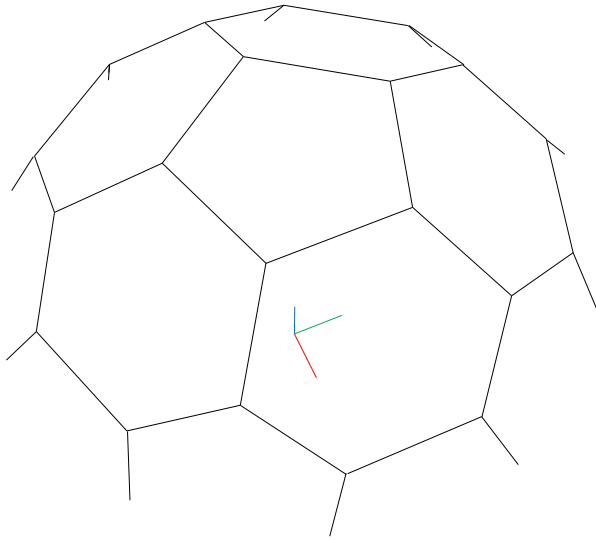
Typical Formian*
generated honeycomb
domes with frequency,
 $m=3$, sweep angle, $A=60$
degrees and (a) $n=6$, (b)
 $n=7$ and (c) $n=8$.

*Software developed at
the University of Surrey, UK

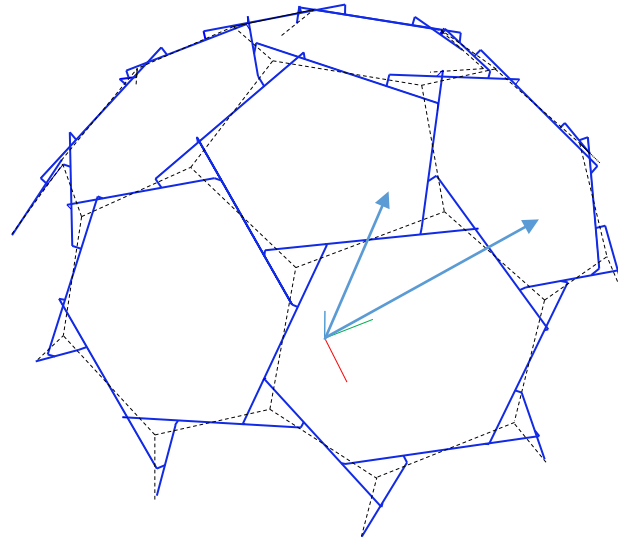
Elementary dome shapes for RSE transformation

- Diamatic domes are another family of lattice domes along with the (a) Ribbed, (b) Schwedler and (c) Lamella type.
- Honeycomb diamatic domes frequently used in practice due to the avoidance of element cluttering near the crown.
- Convenient as RSE configuration processing greatly simplified.

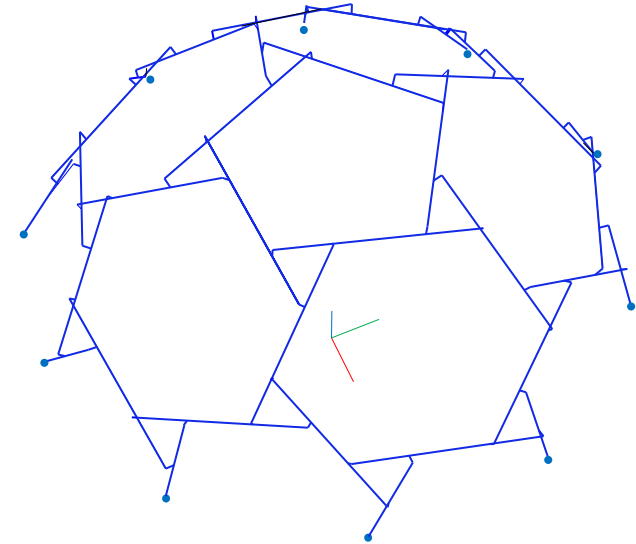
RSE Transformation – Formian and Rhinoceros



(a)



(b)



(c)

Perspective views of (a) elementary dome, (b) rotation method transformation and (c) RSE honeycomb dome using initial rotation angle of 15° .

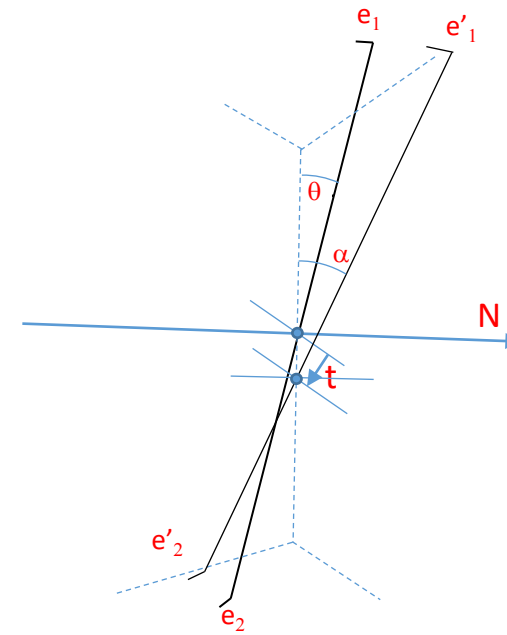
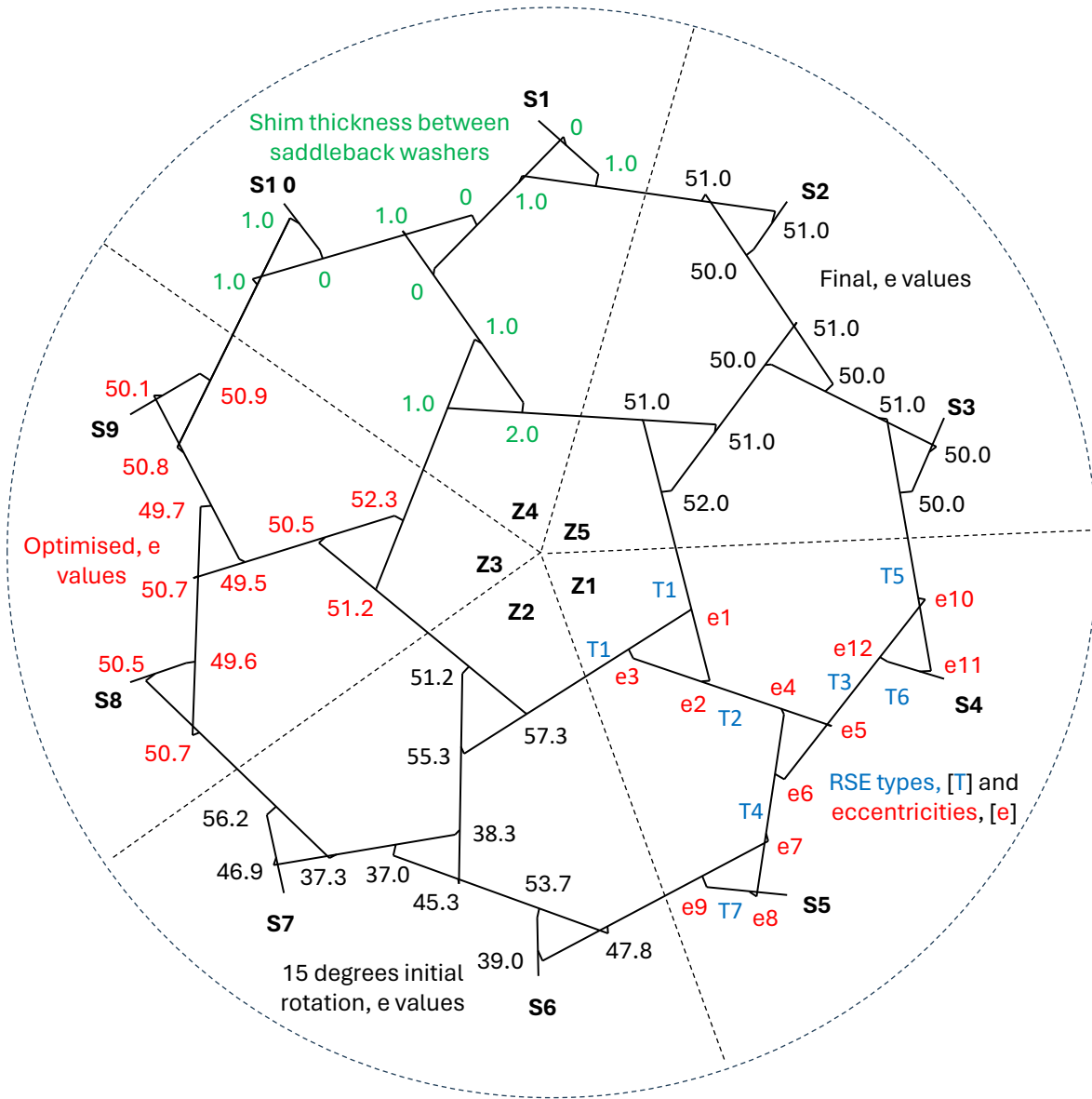
Case Study – RSE Honeycomb Dome

- Study aim was to compare predicted behaviour with monitored behaviour in the laboratory.
- Part 1 of the study was structural modelling.
- Part 2 involved manufacture, construction, loading and monitoring in the laboratory.
- Experimental output would allow modelling calibration.

Transformation Optimisation

Dome construction

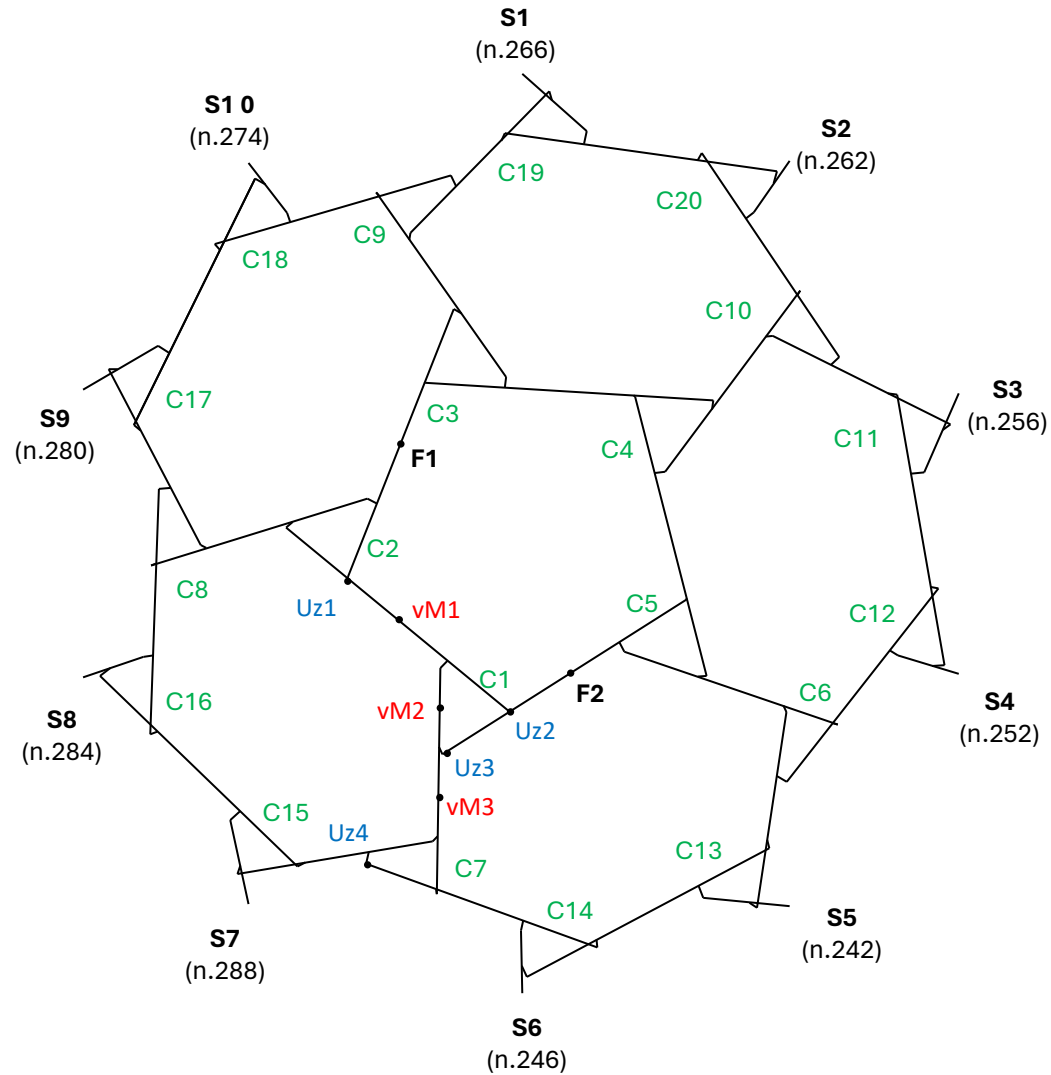
- (i) 48.3mm dia. cylindrical Circular Hollow Section (CHS) tubes.
- (ii) 12mm diameter bolts in 13.0 mm diameter clearance (oversized) holes.
- (iii) Saddleback washers used with a minimum thickness of 0.85 mm for accurate seating and location.
- (iv) Modified rotation method was used to achieve a 50mm (+) 2.5 mm or (-) 0.5 mm target eccentricity.
- (v) RSE dome span and rise was determined as 3066 mm and 894 mm respectively.



Eccentricity Optimisation

RSE Honeycomb dome plan.
 Boundary supports, [S1 - S10].
 Elements types, [T1 - T7].
 Bolted connections eccentricity, [e1 - e12].

Note: all sector zones [Z1 - Z5] identical with symmetry indicated by dotted lines.



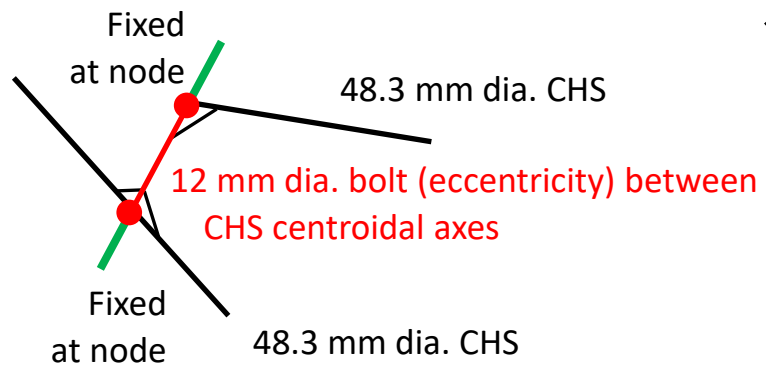
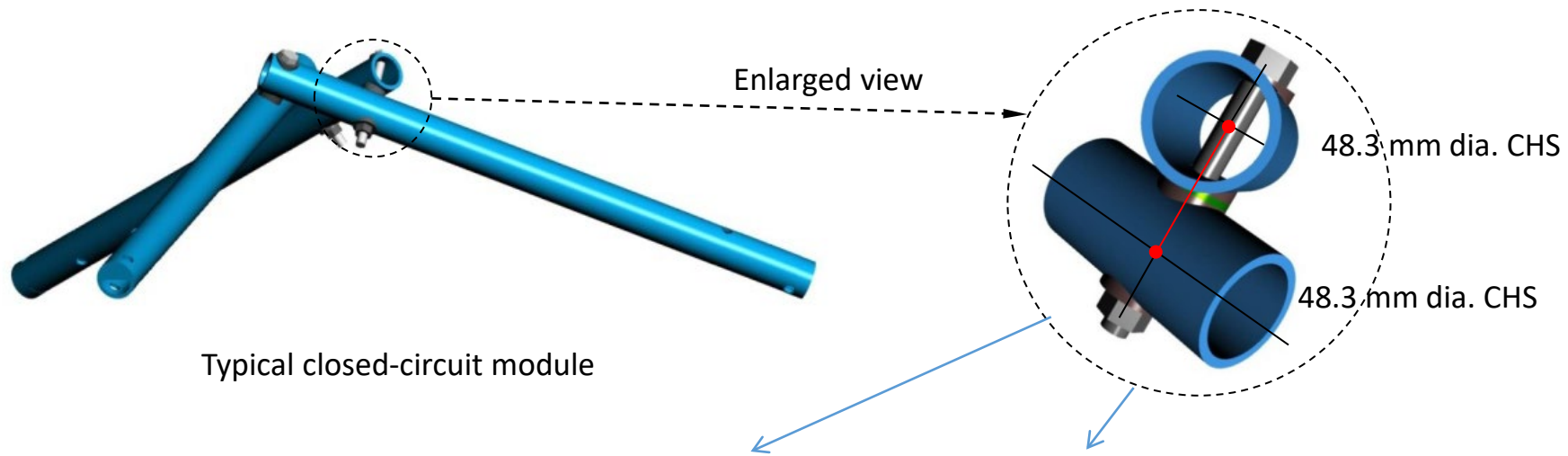
GSA analysis boundary supports node numbering.

Loading positions indicated by [F1 - F2].

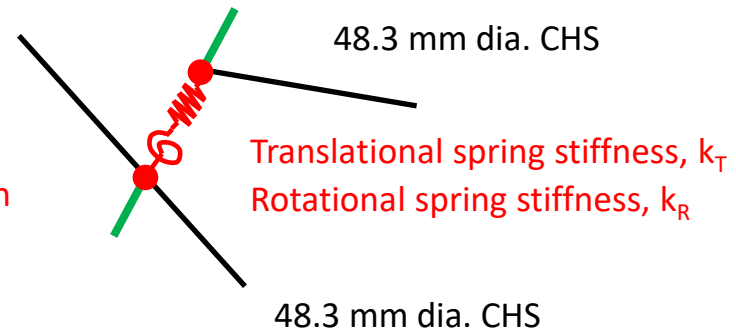
Displacement monitoring, [Uz1 - Uz4].

GSA analysis and lab von Mises stresses monitoring locations, [vM1 - vM3].

Module circuits numbered [C1 - C20].

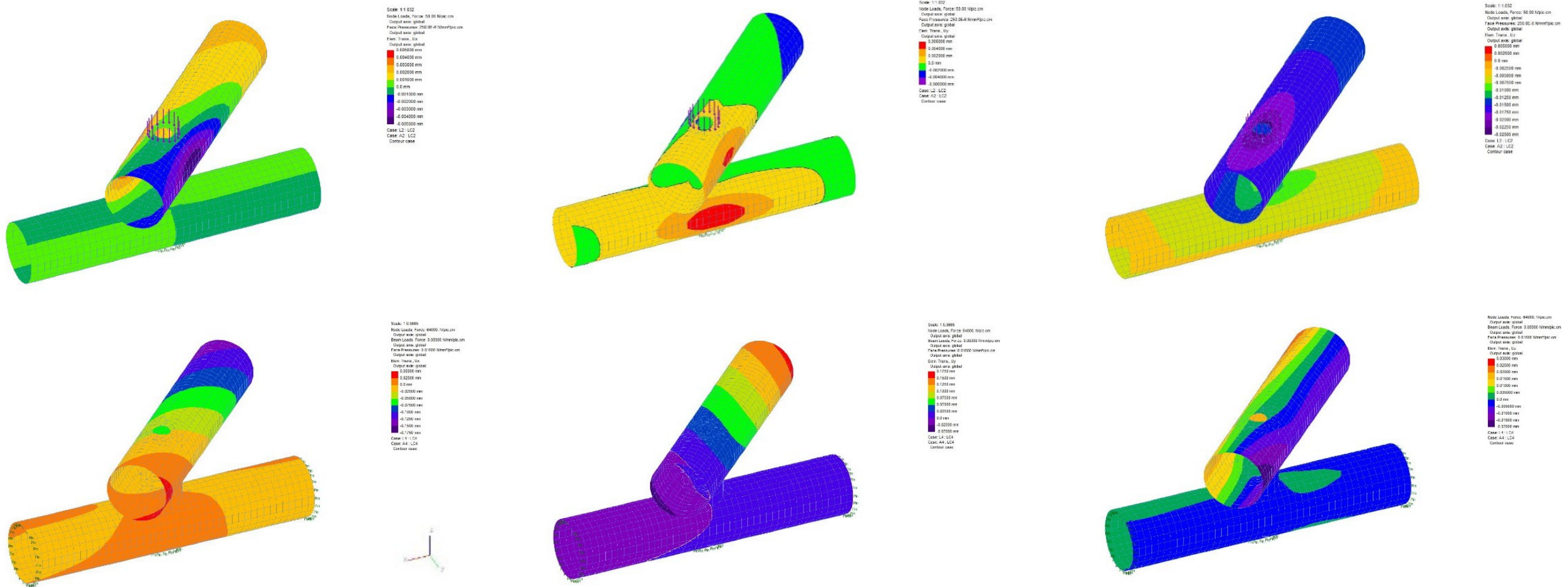


Model 1.



Model 2.

CHS element bolted connections



FEA connection models used to determine translational, k_T and rotational, k_R spring stiffnesses.

		x/xx		y/yy		z/zz	
Spring	Type	Linear/ curve ref.	Stiffness (kN/m) (kNm/rad.)	Linear/ curve ref.	Stiffness (kN/m) (kNm/rad.)	Linear/ curve ref.	Stiffness (kN/m) (kNm/rad.)
Property 1	Translational	Linear	14364	Linear	10231	Linear	15184
Property 2	Rotational	Linear	29.6	Linear	26.9	Linear	266.7

Model 2

Structural Modelling

Boundary supports

- Objective to model the experimental support conditions.
 - (i) Minor geometric self-adjustments would take place within the dome structure when initial loading commenced.
- A range of 8 No possible support leg conditions considered including:
 - (i) May be free to move laterally as they would not be mechanically fixed in position,
 - (ii) Vertical z-direction restraint + horizontal axial stiffness, k_x and k_y applied

Connections

- (i) Model 1 – All fixed both ends
- (ii) Model 2 – All with k_T and k_R spring stiffness.

Model		Boundary	Connection
U	vM	Boundary Support Legs (S1 to S10)	Connections (60)
1	1	All 10 nodes pinned	Model 1: All fixed
2	2	2 nodes pinned (S10 n.274, S1 n.262). 8 nodes horizontal rollers restrained vert. z-dir.	Model 1:
2a	3	3 pinned (S10 n.274, S1 n.262, S6 n.246). 7 nodes horizontal rollers restrained vert. z-dir.	Model 1:
3	4	10 nodes horizontal spring stiffnesses, k_x & k_y restrained vert. z-dir.	Model 1:
4	5	All 10 nodes pinned	Model 2: All with Translational and Rotational spring stiffnesses, k_T & K_R
2	6	2 nodes pinned (S10 n.274, S1 n.262). 8 nodes horizontal rollers restrained vert. z-dir.	Model 2:
5a	7	3 pinned (S10 n.274, S1 n.262, S6 n.246). 7 with horizontal rollers restrained vert. z-dir.	Model 2:
6	8	10 nodes horizontal spring stiffnesses, k_x & k_y restrained vert. z-dir.	Model 2:

Analysis Models

Linear Elastic Analysis

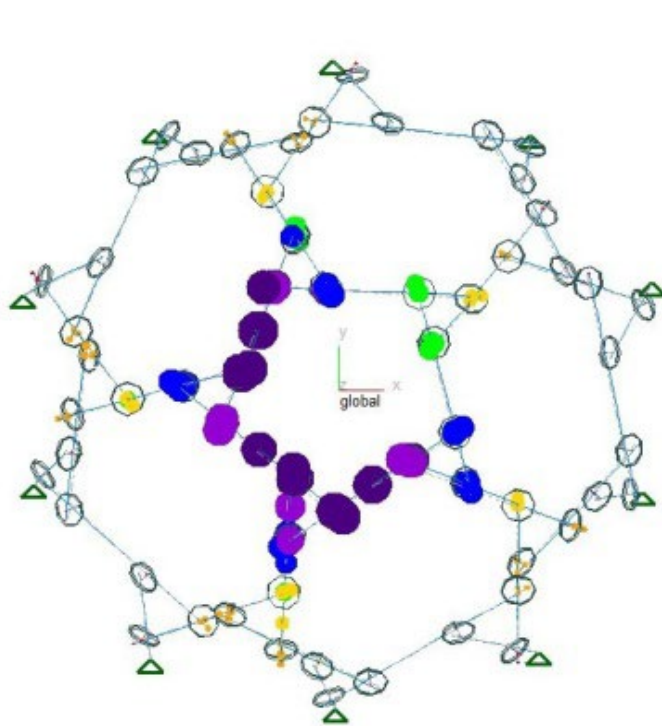
Arup Oasys GSA 8.7 (General Structural Analysis) software with 3-dimensional and finite element capabilities used.

- Two property types defined
 - 48.3 mm diameter 4.0 mm thick grade S355 CHSs
 - 12 mm diameter grade M8.8 bolts used for connecting the CHSs together in closed triangulated circuits.
- Two property types defined the translational and rotational stiffnesses.
- Applied load range of 1 kN to 8 kN
- Varying boundary supports and connections models considered.

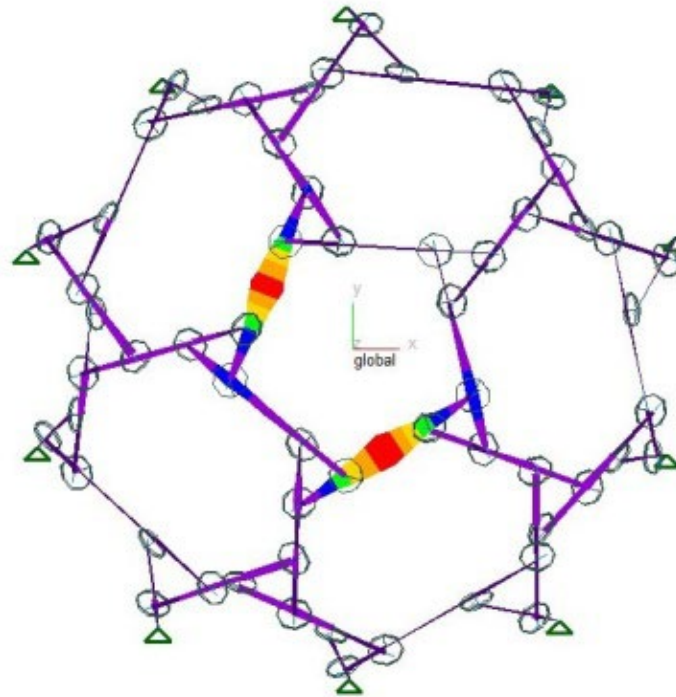
Output.

- Displacements, U_x , U_y and U_z
- Von Mises stresses,

$$\sigma_{VM} = (\sigma_{xx}^2 + 3\tau_{xy}^2 + 3\tau_{xz}^2)^{0.5} \leq \sigma_y \text{ (yield strength of material) monitored}$$



Element list: G1 G2 G3 G4 G5 G6 G7 G9
 Scale: 1:27.0
 Deformation magnification: 25.00
 Translation, Uz: 10.00 mm/pic.cm
 Output axis: global
 0.2670 mm
 -0.7856 mm
 -1.838 mm
 -2.891 mm
 -3.943 mm
 -4.996 mm
 -6.049 mm
 -7.101 mm
 Case: L3 : LC3
 Case: A3 : LC3
 Contour case

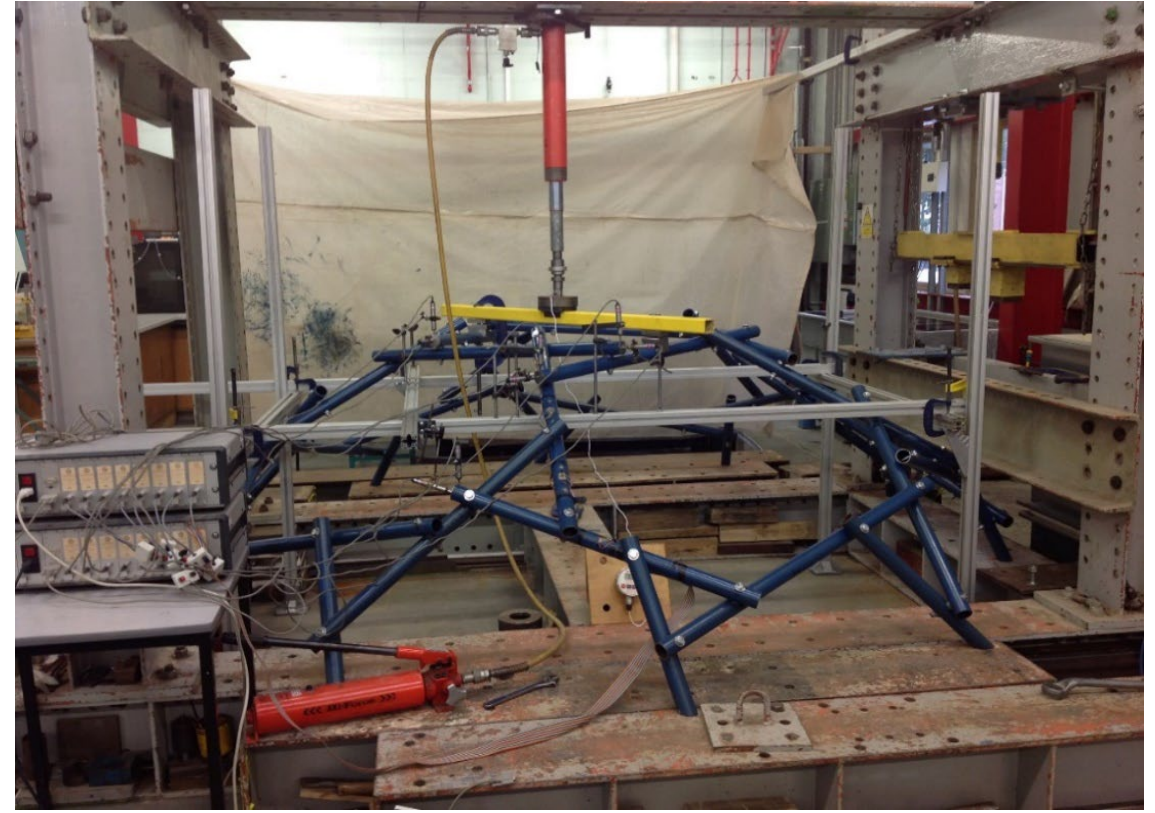


Element list: G1 G2 G3 G4 G5 G6 G7 G9
 Scale: 1:27.0
 Beam Stress, Von Mises: 250.0 N/mm²/pic.
 151.0 N/mm²
 129.5 N/mm²
 107.9 N/mm²
 86.30 N/mm²
 64.73 N/mm²
 43.15 N/mm²
 21.58 N/mm²
 7.985E-6 N/mm²
 Case: L3 : LC3
 Case: A3 : LC3
 Contour case

(Model 5) All supports pinned. All connections with spring stiffnesses.



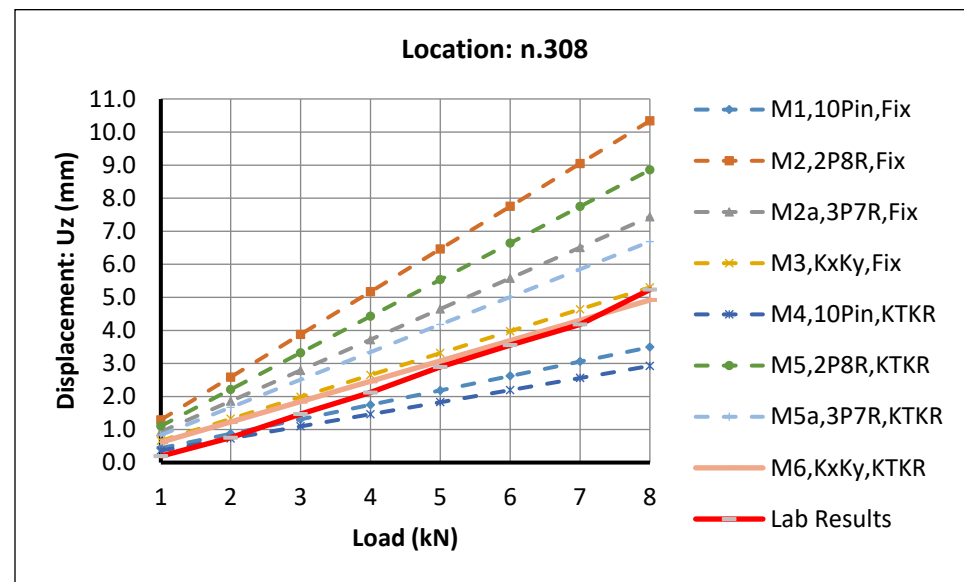
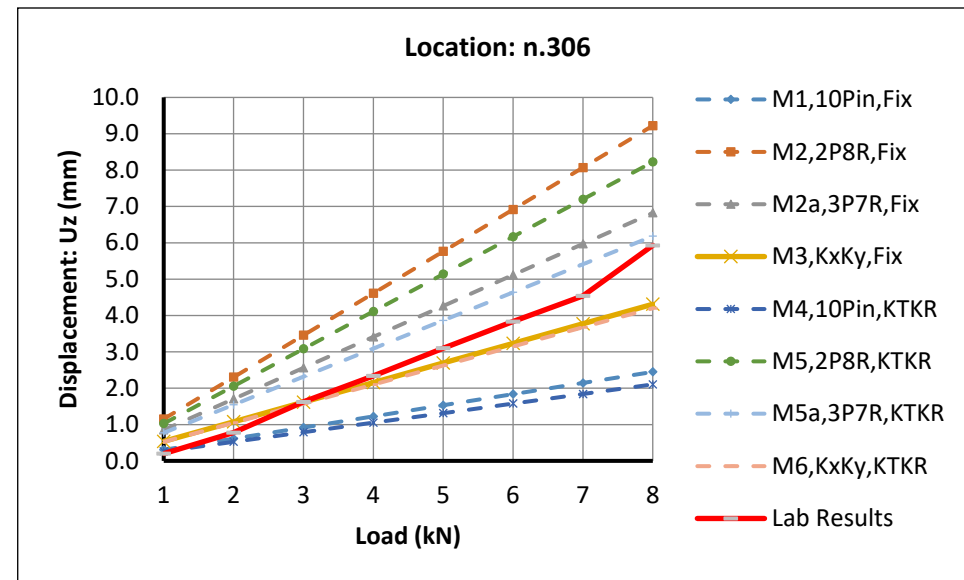
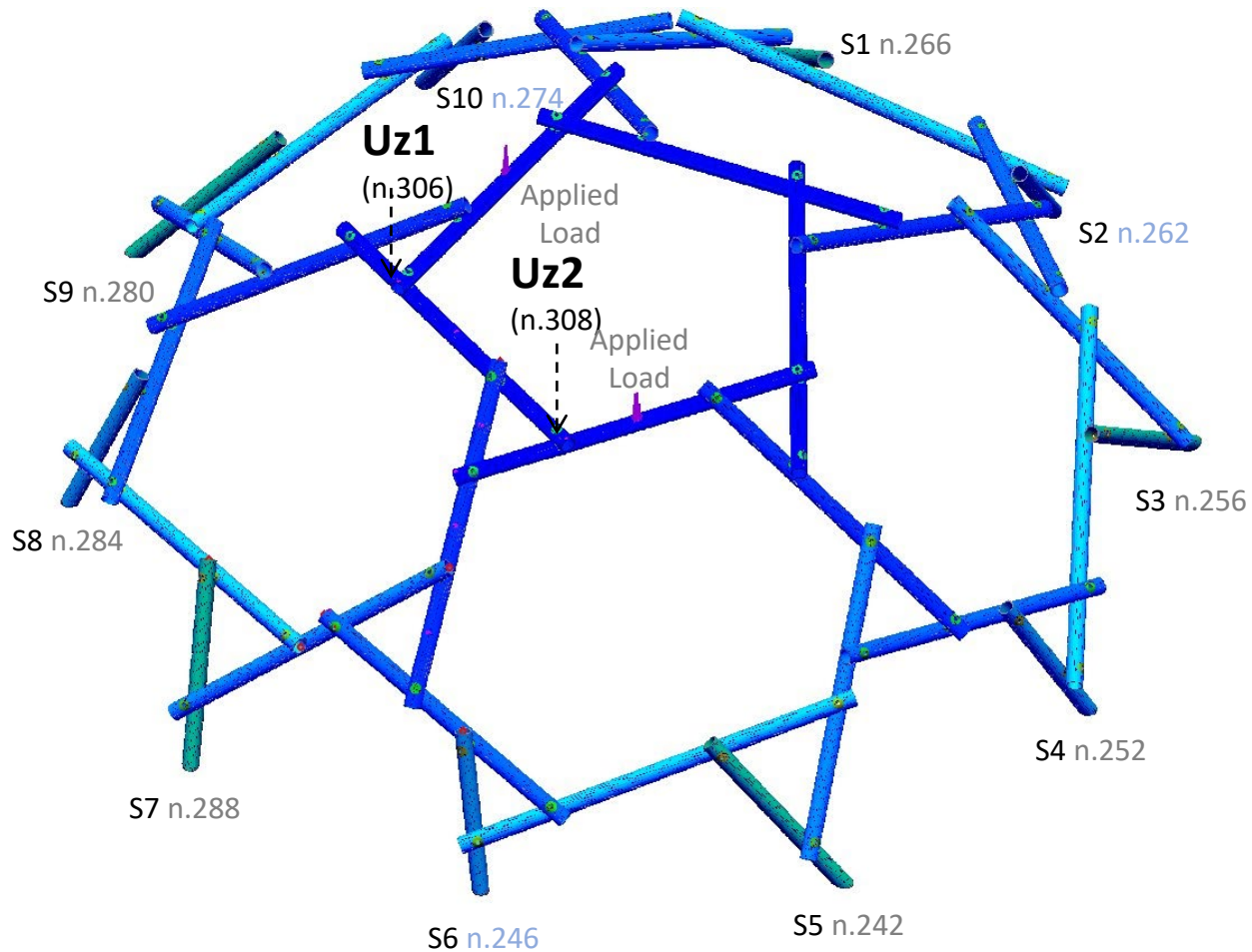
RSE honeycomb dome being constructed.



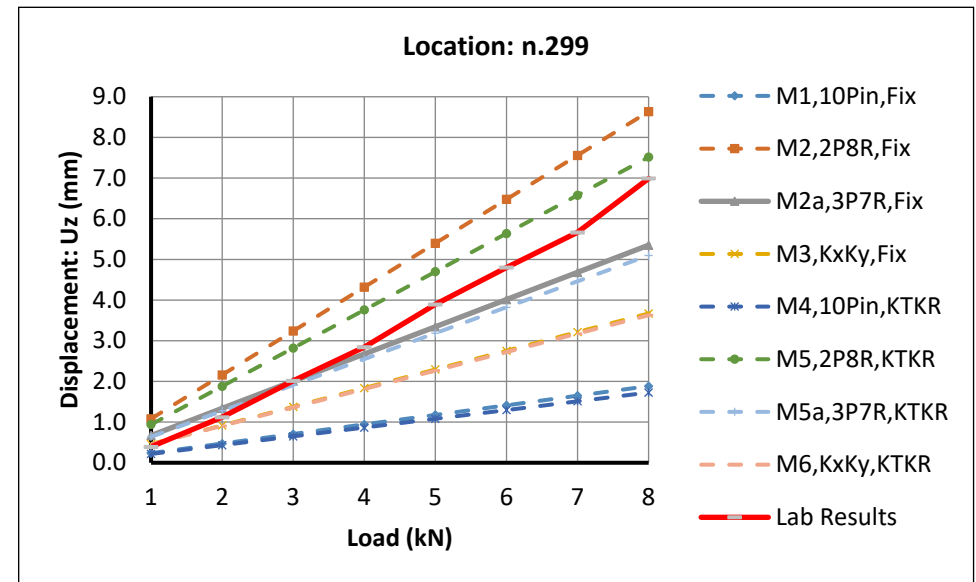
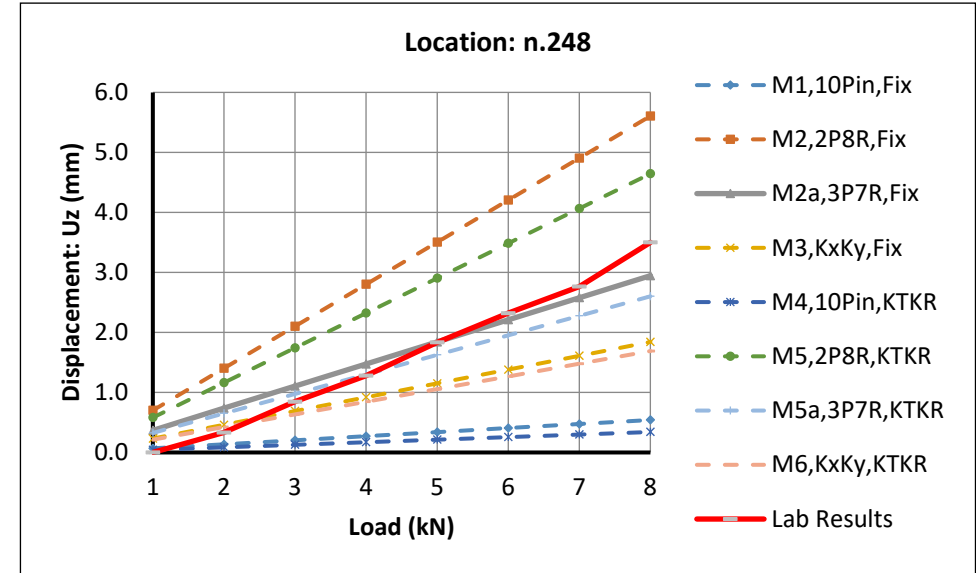
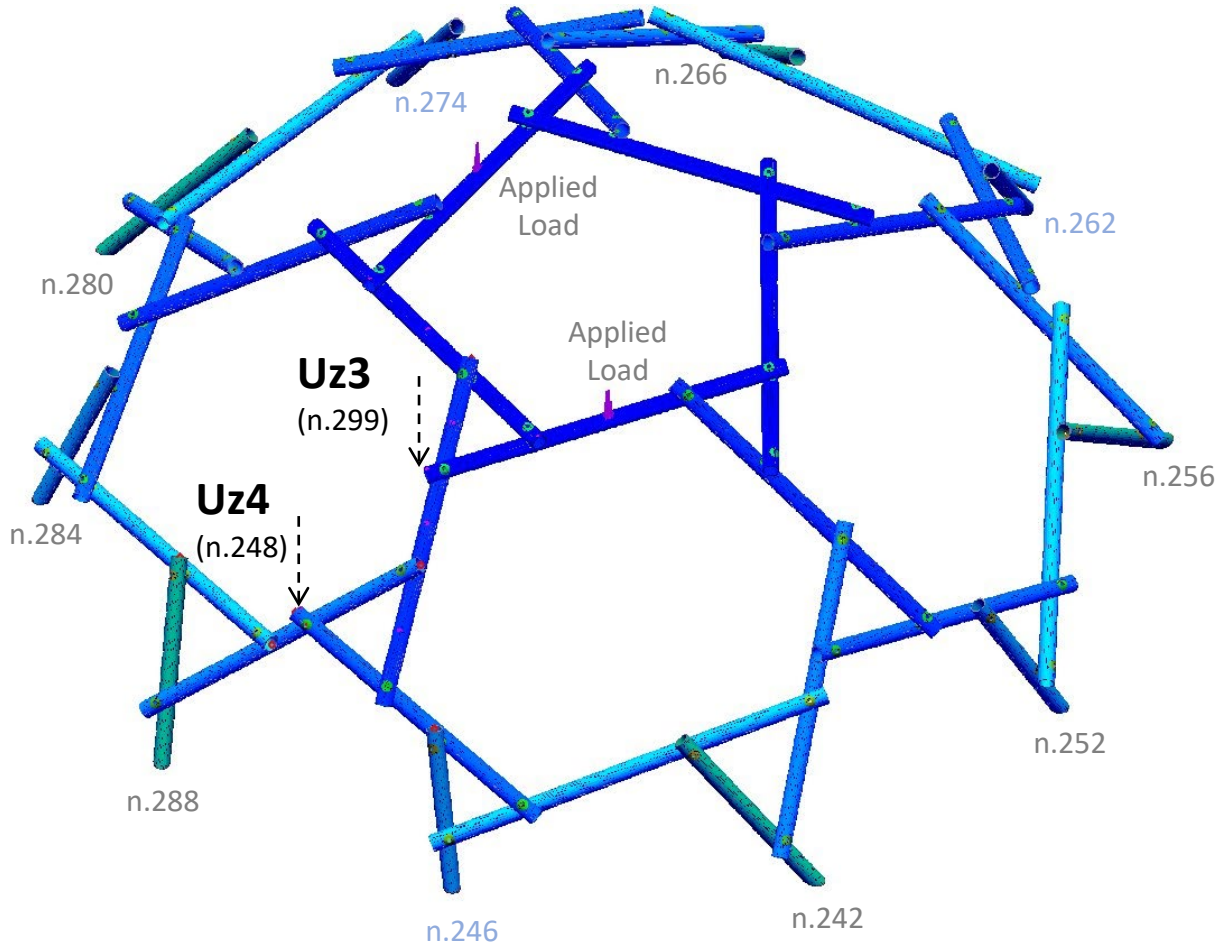
RSE honeycomb dome in test setup.

- Rosette stacked strain gauge matrix orientation on upper tube surfaces at monitoring locations.
- Hydraulic ram loaded spreader beam and CHS tube bearing assembly with LVDT set up for displacement monitoring.

Displacement Monitoring



Displacement Monitoring



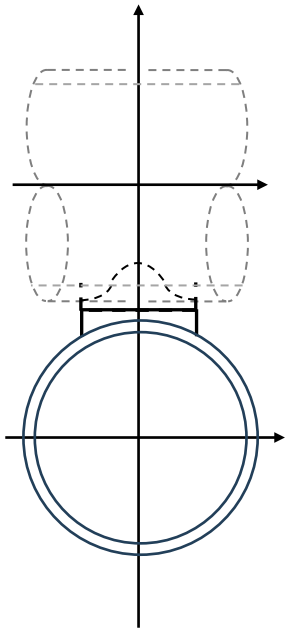
Range of analysis models represent behaviour - further Research required

Important factors in numerical modelling of RSE space structures

- (i) The stiffness of the connection associated with CHSs.
- (ii) CHSs incident angles.
- (iii) Load distribution in connection elements
- (iv) Effective loaded CHS width
- (v) Boundary support stiffness

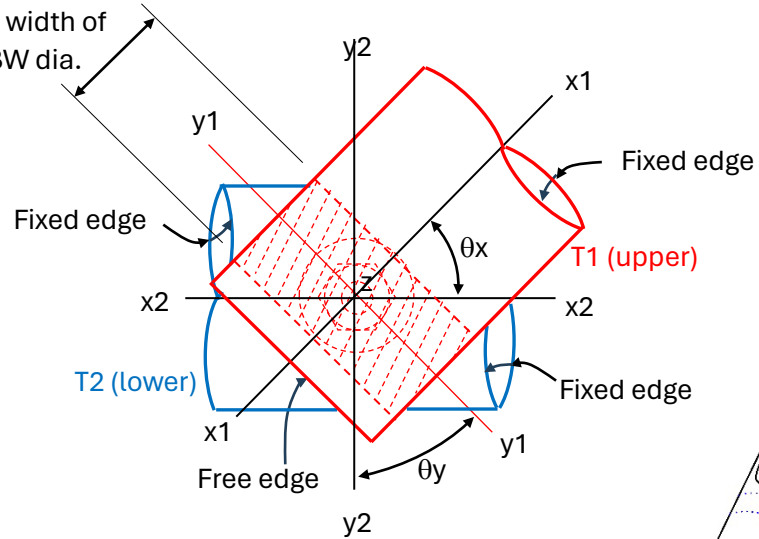
Study widened to determine stiffness of

- (i) RSE CHS welded connections
- (ii) RSE CHS bolted connections



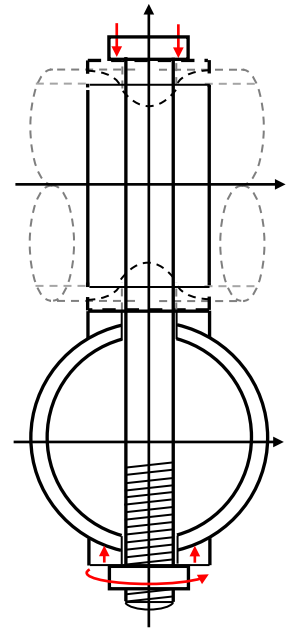
Spring stiffness:
CHS and weld

T1 and T2 effective width of loaded CHS = SBW dia.



Definitions and assumptions CHSs at θ degrees to each other

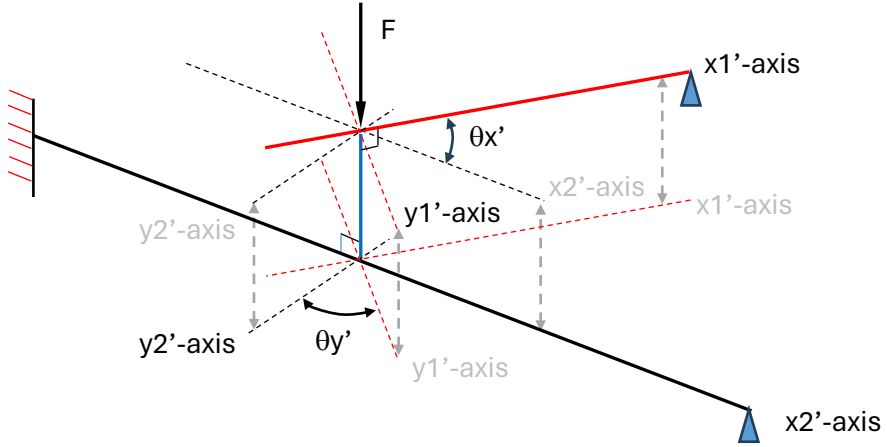
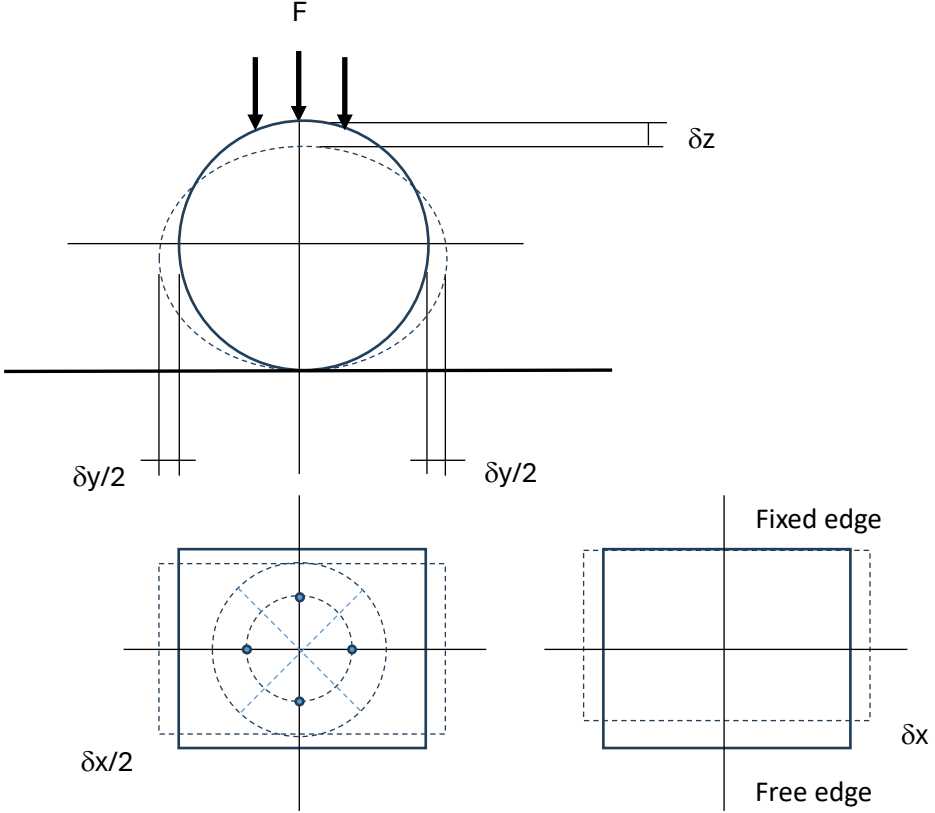
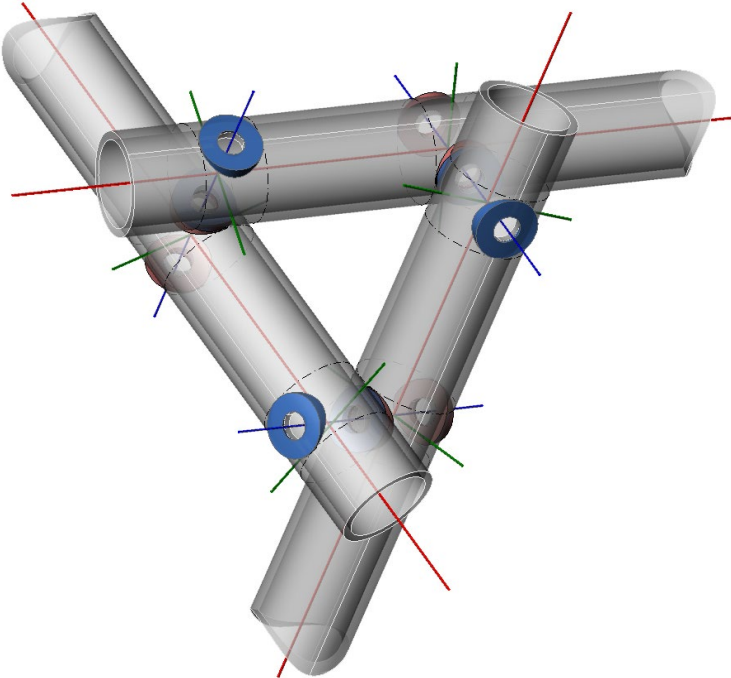
Spring stiffness: Bolt
in tension, CHS in
compression



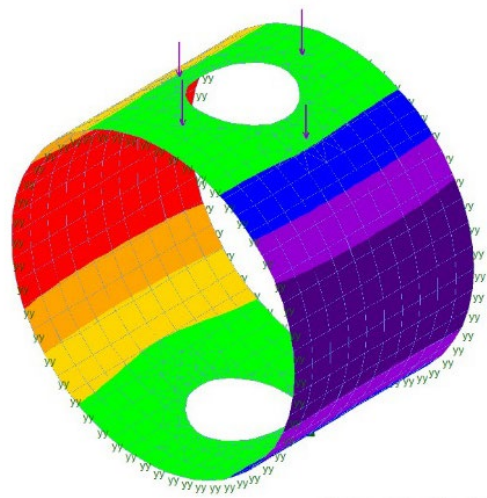
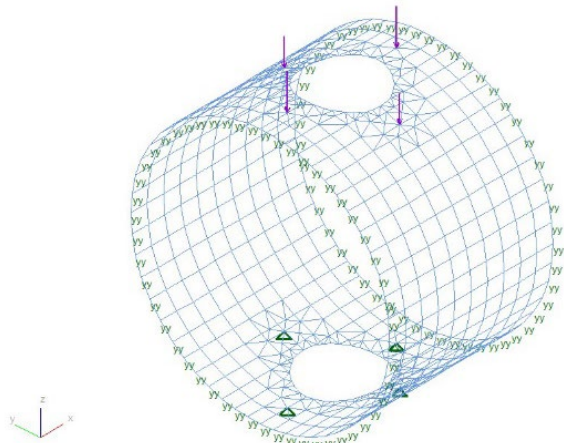
Welded RSE connection with SBW used for locating CHSs.

Bolted RSE connection with SBW and clearance (oversized) holes.

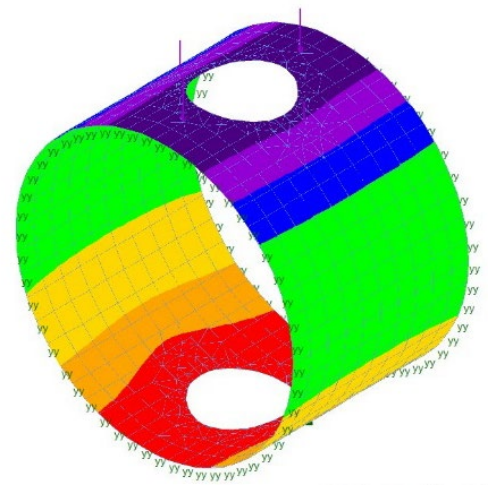
Translational stiffness



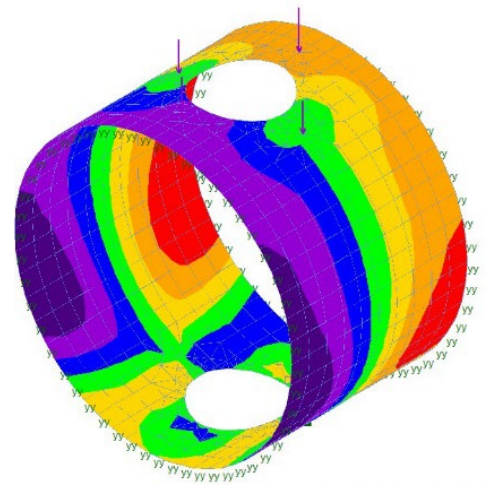
CHS local axes and angles of incidence



Scale: 1.0 4276
 Node Loads, Force: 0.2000 kN/pic
 Output axis: global
 Face Pressures: 0.5000 kN/m²/pic
 Output axis: global
 Elem. Trans., Uy
 Output axis: global
 0.005931 mm
 0.007136 mm
 0.004282 mm
 0.001427 mm
 -0.004282 mm
 -0.007136 mm
 -0.009891 mm
 Case: L2 : LC2 PT Load
 Case: A2 : LC2 PT Load
 Contour case



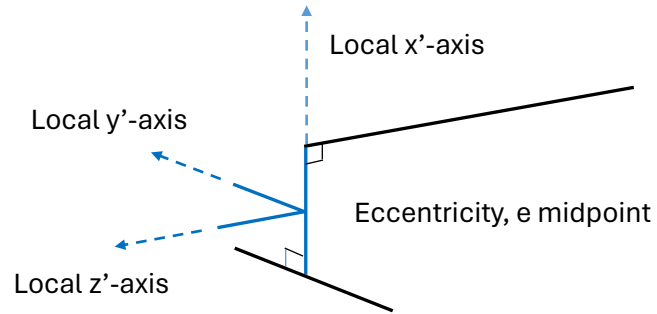
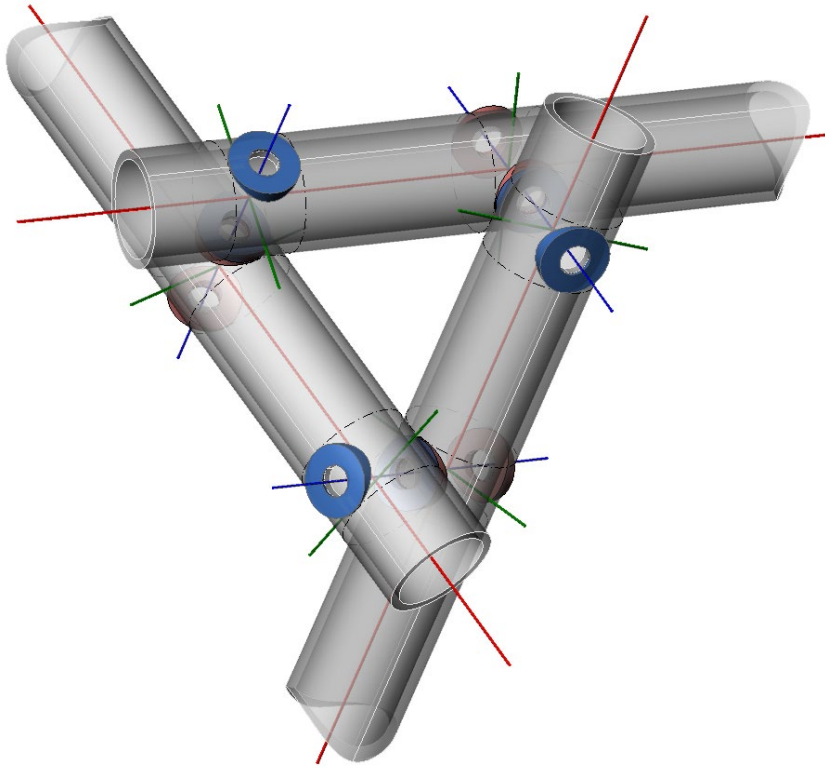
Scale: 1.0 4276
 Node Loads, Force: 0.2000 kN
 Output axis: global
 Face Pressures: 0.5000 kN/m²
 Output axis: global
 Elem. Trans., Uz
 Output axis: global
 0.001242 mm
 -0.001906 mm
 -0.005053 mm
 -0.006200 mm
 -0.01136 mm
 -0.01449 mm
 -0.01764 mm
 -0.02079 mm
 Case: L2 : LC2 PT Load
 Case: A2 : LC2 PT Load
 Contour case



Scale: 1.0 4276
 Node Loads, Force: 0.2000 kN/pic.cm
 Output axis: global
 Face Pressures: 0.5000 kN/m²/pic.cm
 Output axis: global
 Elem. Trans., Ux
 Output axis: global
 88.22E-6 mm
 63.01E-6 mm
 37.81E-6 mm
 12.60E-6 mm
 -12.60E-6 mm
 -37.81E-6 mm
 -63.01E-6 mm
 -88.22E-6 mm
 Case: L2 : LC2 PT Load
 Case: A2 : LC2 PT Load
 Contour case

Translational stiffness – effective width FEA models

Rotational stiffness

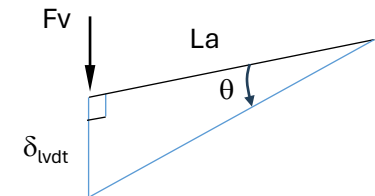
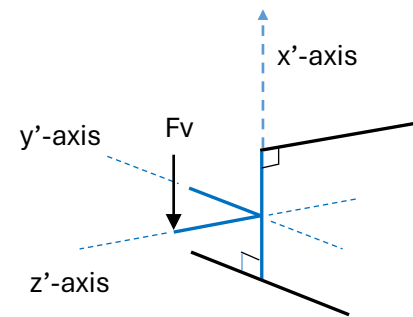
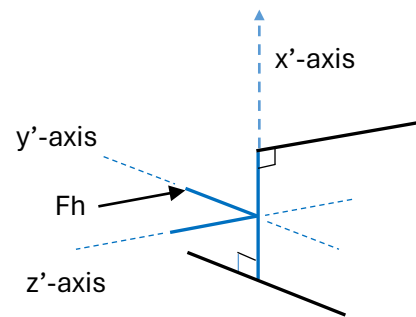
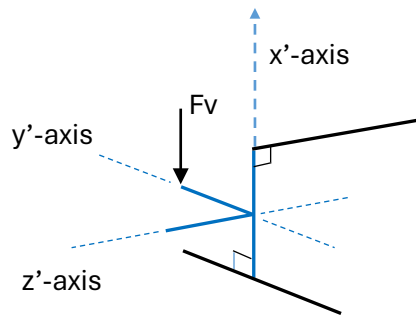


CHS connection rotation stiffness

Fv @ y'-axis, -- Mz'

Fh @ y'-axis, -- Mx'

Fv @ z'-axis, -- My'

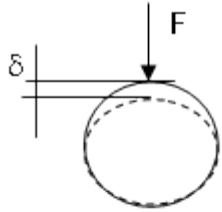


$$My' = (Fv \times La) \text{ (kNm)}$$

$$\theta = \tan^{-1}(\delta_{lvd_t} / La \times \pi / 180) \text{ (radians)}$$

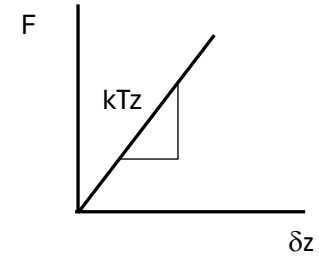
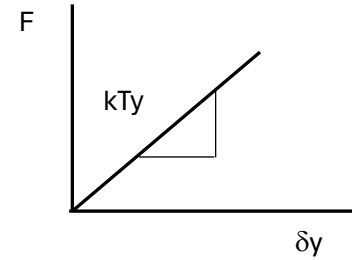
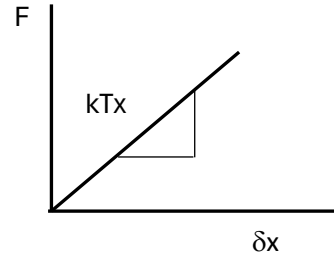
$$k_R = M / \theta$$

Linear springs

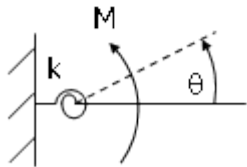


$$kT = F/\delta$$

(kN)/(m)

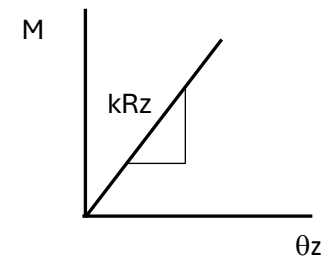
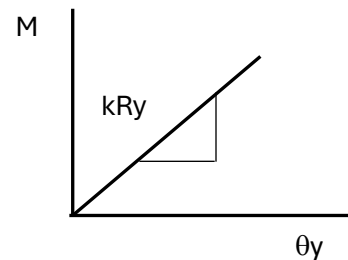
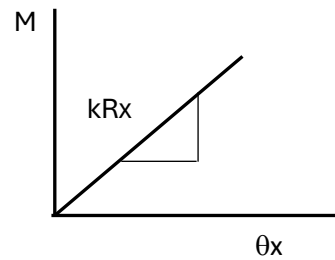


Elastic Translational stiffness, k_{Tx} , k_{Ty} , k_{Tz}



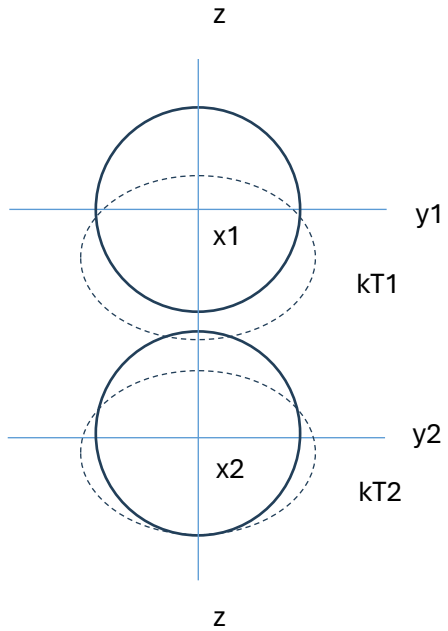
$$kR = M/\theta$$

(kNm)/(Radians)



Elastic Rotational stiffness, k_{Rx} , k_{Ry} , k_{Rz}

Torsional springs

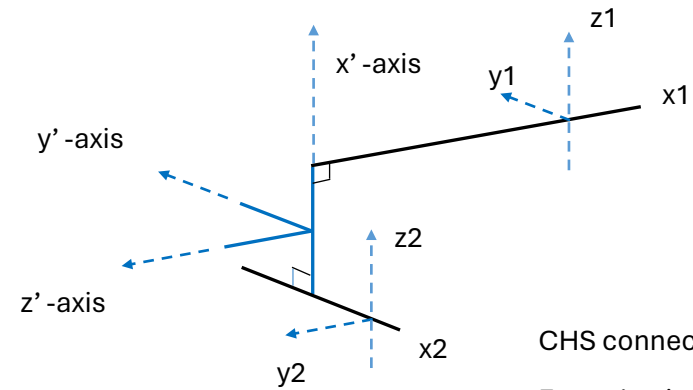


CHS connection translational stiffness

$$k_{Tz} = F_z / \delta z'$$

$$k_{Tx} = F_x / \delta x'$$

$$k_{Ty} = F_y / \delta y'$$



CHS connection rotation stiffness

$$F_v \text{ @ } y'\text{-axis, -- } M_{z'} \quad k_{Rz} = M_{z'} / \theta_{z'}$$

$$F_h \text{ @ } y'\text{-axis, -- } M_{x'} \quad k_{Rx} = M_{x'} / \theta_{x'}$$

$$F_v \text{ @ } z'\text{-axis, -- } M_{y'} \quad k_{Ry} = M_{y'} / \theta_{y'}$$

RSE welded/bolted connection elastic stiffness					
Translational stiffness			Rotational stiffness		
x-axis	y-axis	z-axis	x-axis	y-axis	z-axis
springs in parallel	springs in parallel	springs in series	k_{Rx}	k_{Ry}	k_{Rz}
$K_x = k_1 + k_2$	$K_y = k_1 + k_2$	$1/K_z = 1/k_1 + 1/k_2$			
		$K_z = K_1 K_2 / (k_1 + k_2)$			

Conclusions

Initial Study

- Finite element connection models used to determine spring stiffness used for global analysis.
- Unrealistic high values of stress apparent at connections with assumed full fixity.
- Spring elements developed more representative stresses.

Further investigation identified

- Connection stiffness graphs for varying CHS incident angles
- Effective width of CHS
- Load distribution mechanics
- Welded and bolted connection stiffness differences

References

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Thank you for listening

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