An Erlang multirate loss model supporting elastic traffic under the threshold policy

An Erlang Multirate Loss Model Supporting Elastic Traffic under the Threshold Policy

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Abstract— **In this paper, we propose a multirate teletraffic loss model of a single link with certain bandwidth capacity that accommodates Poisson arriving calls, which can tolerate bandwidth compression (elastic traffic), under the threshold policy. When compression occurs, the service time of new and in-service calls increases. The threshold policy provides different QoS among service-classes by limiting the number of calls of a service-class up to a pre-defined threshold, which can be different for each service-class. Due to the bandwidth compression mechanism, the steady state probabilities in the proposed model do not have a product form solution. However, we approximate the model by a reversible Markov chain, and prove recursive formulas for the calculation of call blocking probabilities and link utilization. The accuracy of the proposed formulas is verified through simulation and found to be very satisfactory.**

***Index Terms*— Poisson Process, Elastic Calls, Threshold Policy, Call Blocking, Recursive Formula.**

I. INTRODUCTION

Multirate loss models based on recursive formulas provide an efficient way for the call-level QoS assessment in communication networks which accommodate elastic traffic. Elastic traffic refers to in-service calls whose bandwidth can be compressed, while their service time increases. The call-level analysis of a link that behaves as a loss system and accommodates multirate elastic traffic is based on the classical Erlang Multirate Loss Model (EMLM) [1],[2].

In the EMLM, a link of capacity C bandwidth units (b.u.) accommodates K service-classes. Service-class k calls (k=1,…,K) follow a Poisson process with arrival rate λ_k and require a peak-bandwidth of b_k b.u. Calls compete for the available link b.u. under the Complete Sharing (CS) policy. According to the CS policy, new calls are blocked only if their required b.u. are more than the available link b.u. Accepted calls remain in the link for an arbitrarily distributed service time [1]. The steady-state probabilities in the EMLM have a Product Form Solution (PFS). The latter leads to an accurate calculation of Call Blocking Probabilities (CBP) via the classical Kaufman-Roberts recursive formula [1],[2] which has led to numerous extensions of the EMLM (e.g., [3]-[19]).

Elastic traffic has been incorporated in the EMLM in [20]. We name the model of [20] Elastic EMLM (E-EMLM). In the E-EMLM, Poisson arriving calls of service-class k have peak and minimum bandwidth requirements of b_k and $b'_{k,\min}$ b.u., respectively. A new service-class k call is accepted in the system with b_k b.u. if the occupied link bandwidth, after the call's acceptance, does not exceed C. If b_k is higher than the available bandwidth, then the system accepts this call by compressing its initial bandwidth b_k together with the bandwidth of all in-service calls. Bandwidth compression of a service-class k call is permitted down to $b'_{k,\min}$. Call blocking occurs if $b'_{k,\min}$ is still higher than the available bandwidth. Bandwidth expansion occurs when a call, with compressed bandwidth, departs from the system. Then, the remaining calls expand their bandwidth in proportion to their peak-bandwidth.

In this paper, we propose the E-EMLM/TH by considering the E-EMLM and modifying the admission mechanism to include the threshold (TH) policy [21]. In the TH policy, the number of in-service calls of service-class k should not exceed a threshold, after the acceptance of a new service-class k call. Otherwise, call blocking occurs. The importance of the TH policy in teletraffic engineering is twofold: i) It analyzes a multirate access tree network which accommodates calls of K service-classes [21]. ii) It provides service-class differentiation in terms of CBP, revenue rates, etc. [22].

Applications of the TH policy are numerous (e.g., [23]-[27]). In [23], the TH policy is applied in a two-tier multirate wireless network. The optimal call admission policy in this network has a two dimensional threshold structure. In [24], [25], the TH policy provides QoS differentiation between new and handoff calls of the same service-class k accommodated in the same cell. In [26], the TH policy provides service differentiation and achieves revenue optimization in a mobile cellular system. In [27], the TH policy is applied in the EMLM and a formula similar to the Kaufman-Roberts formula is proposed for the calculation of link occupancy distribution. We name the model of [27], EMLM/TH.

In the aforementioned papers, in-service calls cannot alter their bandwidth. To the best of our knowledge, this is the first paper that considers the TH policy in a system that services elastic calls. Bandwidth compression destroys reversibility in the proposed model and therefore no PFS exists. However, we resort to an approximate but reversible Markov chain and prove a recursive formula for the determination of the link occupancy distribution and, consequently, CBP and link utilization. The accuracy of the proposed formulas is verified through simulation and found to be very satisfactory.

The remainder of this paper is as follows: Section II, contains three subsections: In subsection II.A, we present the basic assumptions of the proposed model; the bandwidth compression mechanism is described via an example in subsection II.B, while in subsection II.C, we prove the recursive formula for the link occupancy distribution, provide formulas for the various performance measures and show the relationship of the proposed model with other loss models. In Section III, we provide numerical results whereby the new model is compared to the EMLM/TH and E-EMLM and evaluated through simulation. We conclude in Section IV.

II. The Proposed Model (E-EMLM/TH)

A. The system model

Consider a link of capacity C b.u. that accommodates K elastic service-classes. Service-class k $(k=1,...,K)$ calls follow a Poisson process with arrival rate λ_k and request b_k b.u. Bandwidth compression is introduced in the system by allowing the occupied link bandwidth j to virtually exceed C up to T b.u., i.e., $j=0,1,...,T$. Let $\boldsymbol{n}=(n_1,...,n_K)$ be the vector of all in-service calls and $\boldsymbol{b}=(b_1,...,b_K)$ the vector of peak-bandwidth requirements, then $j=\boldsymbol{nb}$.

The decision to accept a new service-class k call in the system is based on the following constraints: a) The number of in-service calls of service-class k, n_k, together with the new call, should not exceed a threshold n_k^*, i.e., $n_k+1\le n_k^*$. Otherwise the call is blocked. This constraint expresses the TH policy. b) If constraint (a) is met then: b1) if $j+b_k\le C$, the call is accepted in the system with b_k b.u. and remains in the system for an exponentially distributed service time with mean μ_k^{-1}. b2) if $T\ge j+b_k>C$ the call is accepted by compressing its b_k together with the bandwidth of all in-service calls of all service-classes.

The compressed bandwidth of the new service-class k call is:

$$b_k^{'}=rb_k=Cb_k/(j+b_k) \tag{1}$$

where $r\equiv r(\boldsymbol{n})=C/(\boldsymbol{nb}+b_k)=C/(j+b_k)$.

In order to keep constant the product *service time by bandwidth per call*, the mean value of the service time of the new service-class k call changes to $1/\mu_k^{'}=(j+b_k)/C\mu_k$.

The compressed bandwidth of all in-service calls becomes equal to $b_i^{'}=Cb_i/(j+b_k)$ for $i=1,...,K$. When all calls have compressed their bandwidth, then $j=C$. Note that the minimum bandwidth that a call of service-class k tolerates is:

$$b_{k,\min}^{'}=r_{\min}b_k=Cb_k/T \tag{2}$$

where $r_{\min}=C/T$ is the minimum proportion of the required peak-bandwidth and is common for all service-classes.

A new service-class k call, with b_k b.u., is blocked if $j+b_k>T$.

When an in-service call, with compressed bandwidth $b_i^{'}$ departs from the system then the rest in-service calls expand their bandwidth to $b_i^{''}$ in proportion to their b_i, as follows:

$$b_i^{''}=\min\left(b_i,\ b_i^{'}+b_ib_k^{'}\Big/\sum_{k=1}^{K}n_kb_k\right) \tag{3}$$

B. A tutorial example

The following example illustrates the compression/expansion mechanism. Let C=4 b.u., T=8. b.u., K = 2, $\lambda_1=\lambda_2=1$ call/time unit, b_1=2 b.u., b_2=4 b.u and $\mu_1^{-1}=\mu_2^{-1}=1$ time unit. In-service calls of the 1st service-class can be at most three, i.e., n_1^*=3. Similarly, let n_2^*=1 for the 2nd service-class. This system has 7 states $\boldsymbol{n}=(n_1,n_2)$ presented in Fig. 1. Let us examine now the cases of a call arrival/departure.

❖ **Call arrival.** A new 2nd service-class call arrives in the system while the state is (n_1,n_2)= (2,0) and $j=4$ b.u. Since $j^{'}=j+b_2=T$= 8 b.u., the call is accepted in the system after bandwidth compression has been applied to these three calls. In the new state, (n_1,n_2)=(2,1), calls compress their bandwidth to: $b_{1,\min}^{'}=r(2,1)b_1=1.0$, $b_{2,\min}^{'}=r(2,1)b_2=2.0$ so that $j=C$. Similarly, the values of service time become $\mu_1^{-1}/r_{\min}=\mu_2^{-1}/r_{\min}=2.0$.

❖ **Call departure.** Let the system be in state (n_1,n_2)= (2, 1) when a 1st service-class call departs from the system. Then, its bandwidth $b_{1,\min}^{'}=1.0$ is shared to the other two calls in proportion to their peak-bandwidth. So, in state (n_1,n_2)=(1,1) we have: $b_1^{'}=4b_1/6=1.333$ b.u., $b_2^{'}=4b_2/6=2.667$ b.u.

Figure 1 shows the state transition diagram of this example. If we consider the states (n_1,n_2): (1, 0), (1, 1), (2, 1) and (2,0) then the Kolmogorov's criterion (*flow clockwise=flow counter-clockwise*) holds [21]. Thus, the Markov chain is irreversible and the E-EMLM/TH has no PFS. To circumvent this problem, we use state-dependent factors $\phi_k(\boldsymbol{n})$, which lead to a reversible Markov chain:

$$\phi_k(\boldsymbol{n})=\begin{cases}1 & ,\ when\ nb\le C\ and\ \boldsymbol{n}\ in\ \boldsymbol{\Omega}\\ \dfrac{x(\boldsymbol{n}_k^-)}{x(\boldsymbol{n})}, & when\ C<nb\le T\ and\ \boldsymbol{n}\ in\ \boldsymbol{\Omega}\end{cases} \tag{4}$$

where $\boldsymbol{\Omega}$ is the state space $\boldsymbol{\Omega}=\{\boldsymbol{n}:0\le\boldsymbol{nb}\le T,\ n_k\le n_k^*,\ k=1,...,K\}$, $\boldsymbol{n}=(n_1,...,n_k,...,n_K)$, $\boldsymbol{n}_k^-=(n_1,...,n_k-1,...,n_K)$ and

$$x(\boldsymbol{n})=\frac{1}{C}\sum_{k=1}^{K}n_kb_kx(\boldsymbol{n}_k^-),\ when\ C<\boldsymbol{nb}\le T,\ \boldsymbol{n}\ in\ \boldsymbol{\Omega} \tag{5}$$

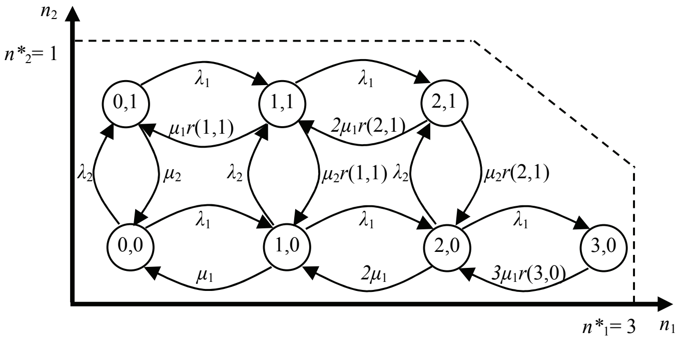

Figure 1. State transition diagram of the tutorial example.

Equation (5) ensures that $\sum_{k=1}^{K} n_k b_k \phi_k(\boldsymbol{n}) = C$, when $T \geq \boldsymbol{nb} > C$.

Figure 2 shows the modified state transition diagram, due to $\phi_k(\boldsymbol{n})$'s, whereby the Kolmogorov's criterion holds.

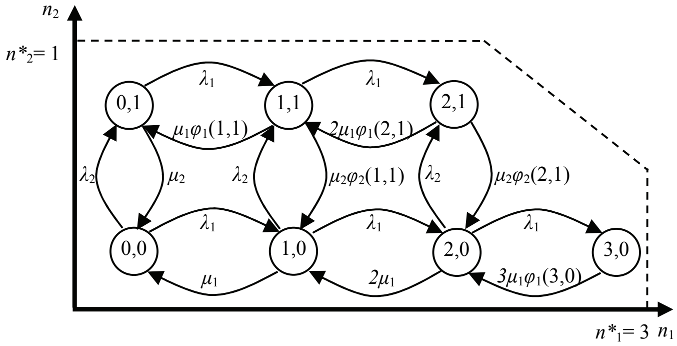

Figure 2. Modified state transition diagram of the tutorial example.

C. The analytical model

The following theorem provides a recursive formula for the calculation of the link occupancy distribution.

Theorem

The link occupancy distribution, $G(j)$, of the modified analytical model satisfies the following recursive formula:

$$G(j) = \frac{1}{\min(C,j)} \sum_{k=1}^{K} a_k b_k \left[G(j-b_k) - T_k(j-b_k) \right], \ for \ j = 1,...,T \qquad (6)$$

where: $G(0) = 1$, $G(y) = 0$ for y<0.

Proof

The global balance equation for state $\boldsymbol{n}$, expressed as *rate into state* $\boldsymbol{n}$ *= rate out of state* $\boldsymbol{n}$, is given by:

$$\sum_{k=1}^{K} \lambda_k P(\boldsymbol{n}_k^-) + \sum_{k=1}^{K} (n_k+1)\mu_k \phi_k(\boldsymbol{n}_k^+) P(\boldsymbol{n}_k^+) = \sum_{k=1}^{K} \lambda_k P(\boldsymbol{n}) + \sum_{k=1}^{K} n_k \mu_k \phi_k(\boldsymbol{n}) P(\boldsymbol{n})$$

where: $\boldsymbol{n}_k^+ = (n_1,...,n_k+1,...,n_K)$ and $P(\boldsymbol{n}), P(\boldsymbol{n}_k^-), P(\boldsymbol{n}_k^+)$ are the probability distributions of states $\boldsymbol{n}, \boldsymbol{n}_k^-, \boldsymbol{n}_k^+$, respectively.

Assume now, the existence of Local Balance (LB) between adjacent states. Then the following LB equations (*rate up=rate down*) can be extracted, for $k = 1,...,K$ and $\boldsymbol{n} \in \boldsymbol{\Omega}$:

$$\lambda_k P(\boldsymbol{n}_k^-) = n_k \mu_k \phi_k(\boldsymbol{n}) P(\boldsymbol{n}) \qquad (7)$$

$$\lambda_k P(\boldsymbol{n}) = (n_k+1) \mu_k \phi_k(\boldsymbol{n}_k^+) P(\boldsymbol{n}_k^+) \qquad (8)$$

Based on the assumption of LB, $P(\boldsymbol{n})$ has the solution:

$$P(\boldsymbol{n}) = G^{-1} \left(x(\boldsymbol{n}) \prod_{k=1}^{K} \frac{a_k^{n_k}}{n_k!} \right) \qquad (9)$$

where $a_k = \lambda_k / \mu_k$ is the offered traffic-load (in erl) of service-class k and $G \equiv G(\boldsymbol{\Omega}) = \sum_{\boldsymbol{n} \in \boldsymbol{\Omega}} \left(x(\boldsymbol{n}) \prod_{k=1}^{K} \frac{a_k^{n_k}}{n_k!} \right)$.

Since j is the occupied link bandwidth, $G(j)$ is defined as:

$$G(j) = \sum_{\boldsymbol{n} \in \boldsymbol{\Omega}_j} P(\boldsymbol{n}), \quad \boldsymbol{\Omega}_j = \{ \boldsymbol{n} \in \boldsymbol{\Omega} : \boldsymbol{nb} = j \} \qquad (10)$$

Consider now two sets: (1) $0 \leq j \leq C$ and (2) $C < j \leq T$. For set (1), we have the EMLM/TH and $G(j)$'s are given by the following accurate and recursive formula [27]:

$$G(j) = \frac{1}{j} \sum_{k=1}^{K} a_k b_k \left[G(j-b_k) - T_k(j-b_k) \right], \ for \ j = 1,...,C \qquad (11)$$

where:

$$T_k(x) := Pr[j = x, \ n_k = n_k^*] \qquad (12)$$

In (12) the fact that $n_k = n_k^*$ implies that $j \geq n_k^* b_k$.

When $C < j \leq T$, we substitute (4) in (7) to have:

$$a_k x(\boldsymbol{n}) P(\boldsymbol{n}_k^-) = n_k x(\boldsymbol{n}_k^-) P(\boldsymbol{n}) \qquad (13)$$

Multiplying both sides of (13) by b_k and summing over k we obtain:

$$x(\boldsymbol{n}) \sum_{k=1}^{K} a_k b_k P(\boldsymbol{n}_k^-) = P(\boldsymbol{n}) \sum_{k=1}^{K} n_k b_k x(\boldsymbol{n}_k^-) \qquad (14)$$

Equation (14), due to (6) is written as:

$$P(\boldsymbol{n}) = \frac{1}{C} \sum_{k=1}^{K} a_k b_k P(\boldsymbol{n}_k^-) \qquad (15)$$

Summing both sides of (15) over $\boldsymbol{\Omega}_j = \{ \boldsymbol{n} \in \boldsymbol{\Omega} : \boldsymbol{nb} = j \}$ and based on (10), we obtain:

$$G(j) = \frac{1}{C} \sum_{k=1}^{K} a_k b_k \sum_{\boldsymbol{n} \in \boldsymbol{\Omega}_j} P(\boldsymbol{n}_k^-) \qquad (16)$$

Since $n_k \leq n_k^*$ then $\sum_{\boldsymbol{n} \in \boldsymbol{\Omega}_j} P(\boldsymbol{n}_k^-) = G(j-b_k) - Pr\left[x = j - b_k, \ n_k = n_k^* \right]$.

Thus, (16) can be written as:

$$G(j) = \frac{1}{C} \sum_{k=1}^{K} a_k b_k \left[G(j-b_k) - T_k(j-b_k) \right] \qquad (17)$$

where $T_k(x)$ is given by (12).

Equations (11), (17) give (6) **(End of Proof)**

Having determined $G(j)$'s we can calculate the CBP of service-class k, B_k, and the link utilization, U, as follows:

$$B_k = \sum_{j=T-b_k+1}^{T} G^{-1} G(j) + \sum_{j=n_k^* b_k}^{T-b_k} G^{-1} T_k(j) \qquad (18)$$

$$U = \sum_{j=1}^{C} j G^{-1} G(j) + \sum_{j=C+1}^{T} C G^{-1} G(j) \qquad (19)$$

where $G=\sum_{j=0}^{T} G(j)$ is the normalization constant.

In (6) and (18) the knowledge of $T_k(j)$ is required. Since $T_k(j)>0$ when $j=n_k^* b_k,...,T-b_k$, we consider two subsets: 1) $n_k^* b_k \le j \le C$ and 2) $C+1 \le j \le T-b_k$.

For the first subset, let a system of capacity $F_k = T - b_k - n_k^* b_k$ that accommodates all service-classes but service-class k. For this system, we define $r_k(j)$ as follows:

$$r_k(j)=\frac{1}{j}\sum_{\substack{i=1\\ i\neq k}}^{K} a_i b_i \left[r_k(j-b_i) - T_i(j-b_i) \right], \quad for \;\; j=1,...,F_k \tag{20}$$

Based on $r_k(j)$'s, we compute $T_k(j)$ via the formula:

$$T_k(j)=\frac{a_k^{n_k^*}}{n_k^*!} r_k(j-n_k^* b_k) \tag{21}$$

For the second subset, $T_k(j)$ can be determined by:

$$T_k(j)=\frac{a_k^{n_k^*}}{n_k^*!}\sum_{\boldsymbol{n}\in\boldsymbol{\Omega}'} x(\boldsymbol{n}) \prod_{\substack{i=1\\ i\neq k}}^{K} \frac{a_i^{n_i}}{n_i!} \tag{22}$$

where: $\boldsymbol{\Omega}' = \left\{ \boldsymbol{n}\in\boldsymbol{\Omega}' : n_k^* b_k + \sum_{i=1, i\neq k}^{K} n_i b_i = j, \; C+1 \le j \le T-b_k \right\}$.

To show the relationship of the proposed model with other multirate loss models, we distinguish three cases:

a) If we do not consider the TH policy in the E-EMLM/TH then $G(j)$'s are given by the E-EMLM, [20]:

$$G(j)=\frac{1}{\min(C,j)}\sum_{k=1}^{K} a_k b_k G(j-b_k), \quad for \;\; j=1,...,T \tag{23}$$

The CBP of service-class k, B_k, is given by:

$$B_k = \sum_{j=T-b_k+1}^{T} G^{-1} G(j) \tag{24}$$

The link utilization can be determined by (19).

b) If $C=T$ and the TH policy is considered, then the EMLM/TH occurs [27]. The CBP of service-class k, B_k, and the link utilization U are given by:

$$B_k = \sum_{j=C-b_k+1}^{C} G^{-1} G(j) + \sum_{j=n_k^* b_k}^{C-b_k} G^{-1} T_k(j) \tag{25}$$

$$U=\sum_{j=1}^{C} j G^{-1} G(j) \tag{26}$$

c) If $C=T$ and we do not consider the TH policy, then the E-EMLM/TH coincides with the EMLM. In that case, $G(j)$'s are given by the Kaufman-Roberts recursion [1], [2]:

$$G(j)=\frac{1}{j}\sum_{k=1}^{K} a_k b_k G(j-b_k), \quad for \;\; j=1,...,C \tag{27}$$

The CBP of service-class k is given by:

$$B_k = \sum_{j=C-b_k+1}^{C} G^{-1} G(j) \tag{28}$$

while the link utilization can be determined by (26).

III. Numerical Results - Evaluation

In this section, we present an application example of the proposed model (E-EMLM/TH) and the models of [20], [27] (E-EMLM and EMLM/TH, respectively). Through the proposed model we obtain analytical CBP and link utilization results, and compare them with the corresponding simulation results, in order to reveal the accuracy of the E-EMLM/TH. The simulation model is based on the bandwidth compression mechanism described by $r(\boldsymbol{n})$'s. Simulation results, based on SIMSRIPT III [28], are mean values of 7 runs. Each run is based on the generation of five million calls. To account for a warm-up period, the blocking events of the first 5% of these calls are not considered in the results. Since reliability ranges are very small, they are not presented in the figures that follow.

Consider a link of capacity C=70 b.u. and three values of T: 1) $T=C$=70 b.u., 2) T=75 b.u. and 3) T=80 b.u. The link accommodates the following three service-classes:

1st service-class: a_1 = 5 erl, b_1 = 2 b.u., $n_1^* = n_{1,\max}$ = 25

2nd service-class: a_2 = 1.5 erl, b_2 = 5 b.u., $n_2^* = n_{2,\max}$ = 11

3rd service-class: a_3 = 1.0 erl, b_3 = 9 b.u., $n_3^* = n_{3,\max}$ = 6

In the x-axis of Figs. 5-11, traffic loads a_1, a_2 and a_3 increase in steps of 1.0, 0.5 and 0.25 erl, respectively. In this way, Point 1 represents the vector (a_1, a_2, a_3) = (5.0, 1.5, 1.0) while Point 7 is (a_1, a_2, a_3) = (11.0, 4.5, 2.5).

In Figs. 5-7, we consider the proposed E-EMLM/TH and present the analytical and simulation CBP results of all service-classes, for all values of T. For comparison, we present the corresponding analytical results of the EMLM/TH. Based on Figs. 5-7, we see that: i) the results obtained by the proposed formulas are close to the simulation results. ii) The bandwidth compression mechanism reduces CBP of all service-classes. iii) The analytical results of the EMLM/TH fail to approximate the simulation results of the E-EMLM/TH.

In Fig. 8, we present the corresponding link utilization results (in b.u.). The link utilization is higher when T= 80 b.u., a result that is expected since this value of T achieves the highest CBP reduction (compared to T=70 or 75 b.u.).

In Figs. 9-11, we consider the E-EMLM/TH together with the E-EMLM and present the analytical CBP results of all service-classes for T=75 b.u. and $n_{3,\max}$ = 3, 4 and 5 calls. The existing E-EMLM fails to approximate the CBP results obtained by the E-EMLM/TH, in the cases of $n_{3,\max}$ = 3, 4. The fact that the two models give quite close CBP results for $n_{3,\max}$ = 5 is explained as follows: Assuming that only calls of the 3rd service-class exist in the link then the theoretical max. number of the 3rd service-class calls is 8 (each of which occupies (70/75)*9=8.4 b.u.). Approaching this value makes the E-EMLM/TH behave as the E-EMLM. We also see that the increase of $n_{3,\max}$ results in the CBP increase for the 1st and 2nd service-classes (Fig. 9 and Fig. 10, respectively) and the decrease of CBP for the 3rd service-class (Fig. 11).

IV. CONCLUSION

In this paper we propose a multirate loss model where Poisson arriving calls of different elastic service-classes compete for the available link bandwidth under the threshold policy. Calls can tolerate bandwidth compression and expansion. The analysis of the proposed model leads to approximate but recursive formulas for the calculation of CBP and link utilization. Simulation results verify the accuracy of the proposed model. In addition, numerical results show the necessity of the proposed model, since existing models fail to approximate the results obtained by the proposed model.

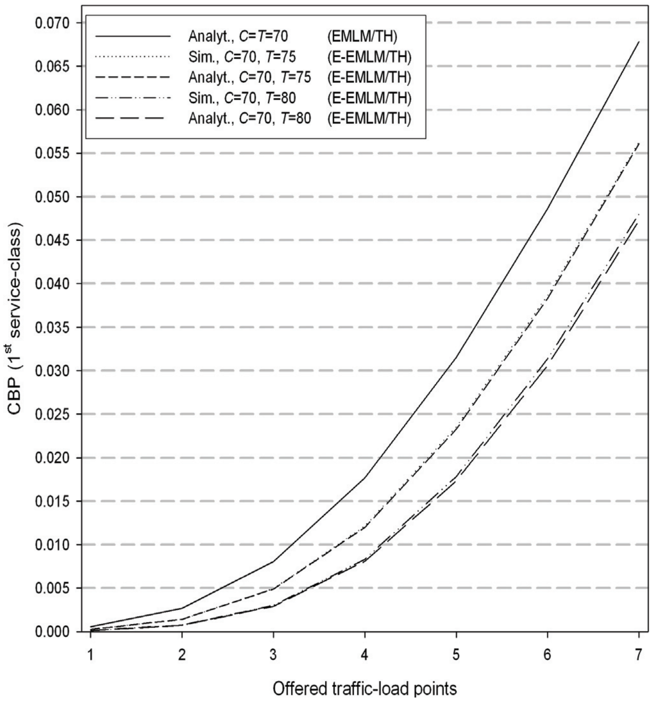

Figure 5. CBP of the 1st service-class, when $n_{3,max}$=6.

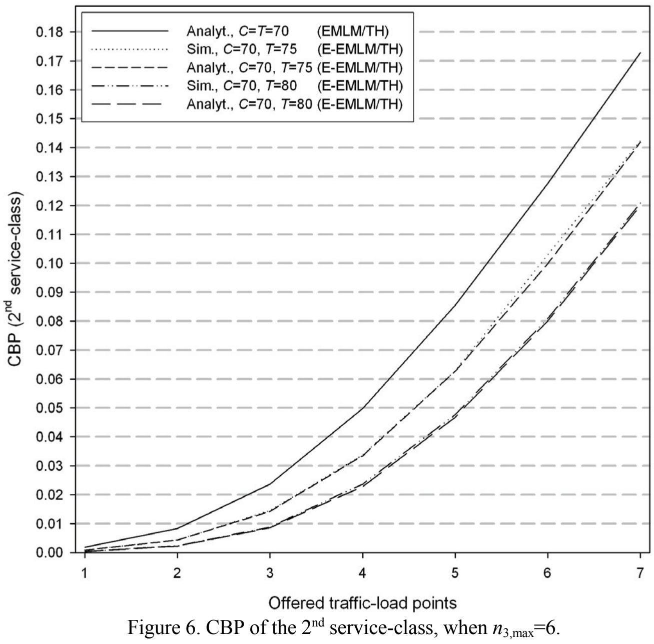

Figure 6. CBP of the 2nd service-class, when $n_{3,max}$=6.

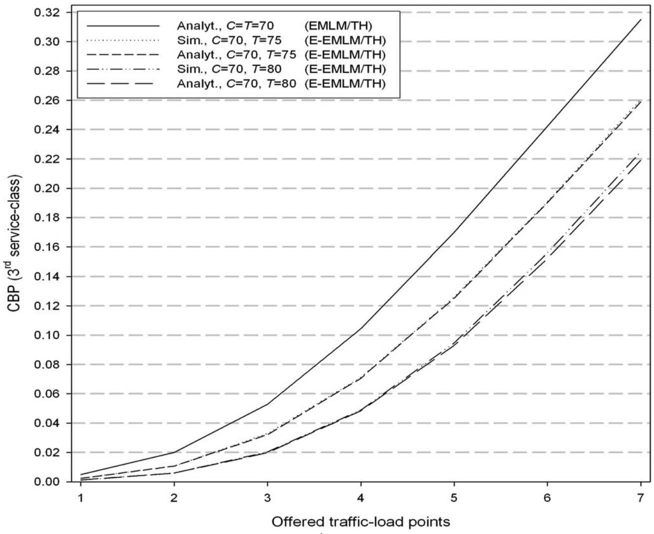

Figure 7. CBP of the 3rd service-class, when $n_{3,max}$=6.

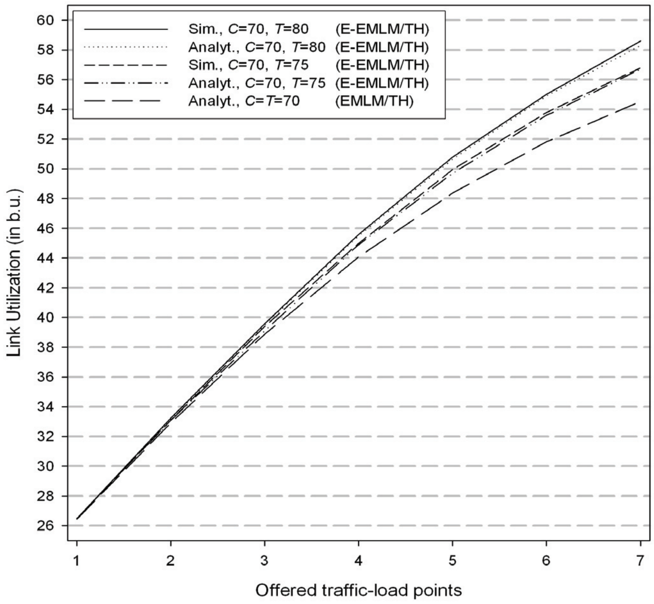

Figure 8: Link Utilization, when $n_{3,max}$=6.

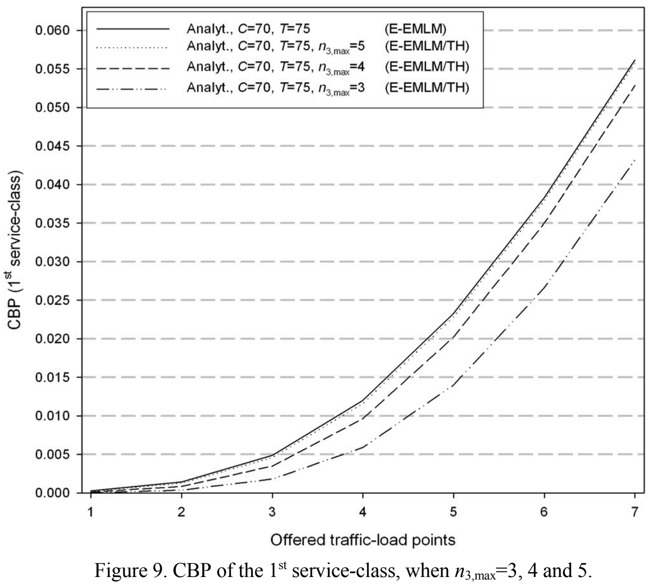

Figure 9. CBP of the 1st service-class, when $n_{3,max}$=3, 4 and 5.

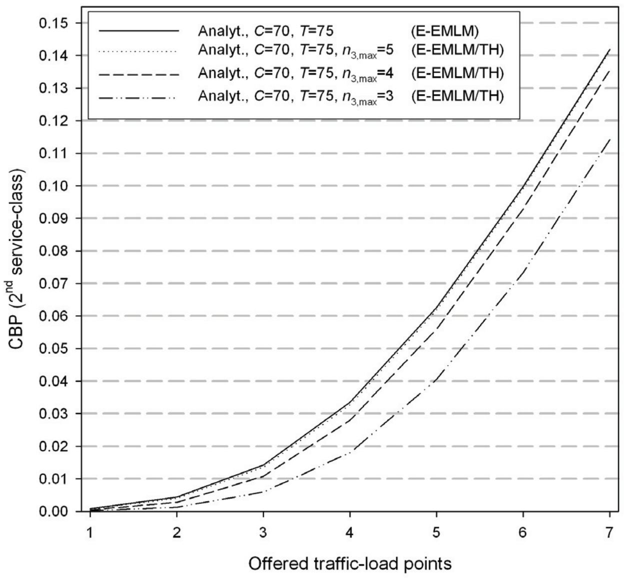

Figure 10. CBP of the 2nd service-class, when $n_{3,max}$=3, 4 and 5.

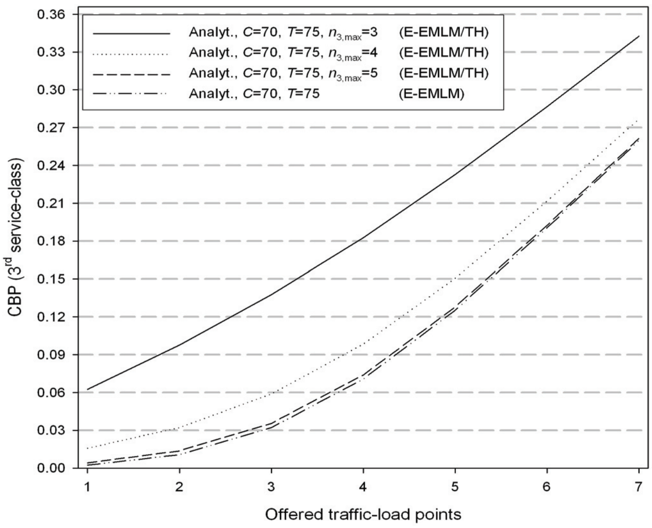

Figure 11. CBP of the 3rd service-class, when $n_{3,max}$=3, 4 and 5.

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