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## SERVICEABILITY BEHAVIOR OF NORMAL AND HIGH-STRENGTH REINFORCED CONCRETE T-BEAMS

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### Abstract

*Serviceability behavior of Normal Strength Concrete (NSC) and High Strength Concrete (HSC) T-beams was experimentally evaluated. The crack pattern was observed, the effect of flange dimensions (breadth and thickness) on the crack pattern and load-deflection response was evaluated experimentally for 10 beams comprising the two studied groups, NSC and HSC T-beams. The short-term deflections were measured experimentally and predicted empirically under mid-span concentrated loading. It was found that increasing the flange width and thickness resulted in higher loads and lower deflections under service loads to a different extent. Prior to failure, the increment in the maximum loads was up to 22% while the deflection reduced by 31% for NSC and 23% for HSC beams. The available equations for determining the effective moment of inertia ( $I_e$ ) were reviewed and used in predicting the  $I_e$  of the cracked beam. The results were compared with the experimental values ( $I_{exp}$ ). The  $I_e$  showed a noticeable difference, especially for the HSC T-beams. New equations were proposed in which the tensile reinforcement ratio was considered. Compared with the other available equations, the proposed equations demonstrated a better agreement and repeatability of predicting experimental results studied herein. In addition, the proposed equations were used to predict the  $I_e$  for experimentally tested T-beams available in the literature. The proposed models showed a high degree of accuracy.*

**Keywords:** T-beams, high strength concrete, normal strength concrete, short term deflection, crack pattern, load-deflection, effective moment of inertia.

### Introduction

In the design of reinforced concrete structures, a designer must satisfy not only the strength requirements but also the serviceability requirements. Therefore, the control of deformation becomes more important. To ensure serviceability criteria, it is necessary to accurately predict the cracking and deflection of reinforced concrete structures under service loads. For accurate determination of the member deflections, cracked members in reinforced concrete structures need to be identified and their effective flexural and shear rigidities determined [1]. Over the years, many researchers studied the deflection of reinforced concrete (RC) flexural members, and consequently various methods have been proposed for predicting the deflection under both short and long-term loadings for NSC and HSC rectangular beams [2, 3]. Qin et al [4] studied the shear behaviour of HSC T-beams strengthened by CFRP advanced composites. González and Ruiz [5] studied the influence of flanges on the shear-carrying capacity of reinforced concrete T-beams without web reinforcement. Effective moment of

inertia of entire spans at service load is one of the empirical and simple methods used in calculating the short-term deflection. The main factors affecting the short-term deflections of a beam are the span length, type and magnitude of loading, material properties, sectional dimensions, and extent of cracking [2, 6].

Branson [7] developed the well-known equation for the average effective moment of inertia  $I_e$  over the entire length of a simply supported, uniformly loaded, rectangular beam. Most of the codes of practices adopted this equation for predicting the short-term deflection [8-11]. Al-Zaid et al. [12] modified Branson's equation to include the effect of loading type by considering the variations in the cracked length for each type of loading. Al-Shaikh and Al-Zaid [13] suggested simple procedures to account for the effect of the reinforcement ratio on the values of the effective moment of inertia of cracked NSC reinforced rectangular beams. In addition, some researchers further developed the equation to include material properties of GFRP bars [14, 15]. Mathematical models were developed for predicting the effective moment of inertia and the deflection of NSC reinforced rectangular and T-beams taking into consideration the effect of loading type for different longitudinal reinforcement ratios [2, 3, 16]. Several studies can be found in the literature on the short-term and long-term deflections of the HSC rectangular beams [16, 17]. However, limited studies considered the short-term and long-term deflections of the HSC T-beams [18].

This study aims to investigate the experimental behaviour (crack pattern and load-deflection relationships) of the NSC and HSC T-beams under service loads prior to failure. In addition, the effective moment of inertia and the short-term deflection of the studied beams will be evaluated experimentally and predicted theoretically under service loading. The significance of flange dimensions (breadth and depth) on the effective moment of inertia of cracked beam sections will be also quantified experimentally and predicted empirically.

## Experimental Programme

### *Specimen details*

The experimental work involved casting and testing of 10 simply supported RC T-beams. Two series of beams were used, one for NSC beams and the other for HSC beams. Five different flange dimensions were used for each series. Fig. 1 and Table 1 show the details of specimens.

The tested beams had a clear span of 1500 mm. The beams were tested under mid-span concentrated loads and a shear span-to depth ratio ( $a/d$ ) of 3.33. All beams were reinforced with  $2\Phi 16$  bars as a main reinforcement,  $2\Phi 10$  for the top bars (yield strength of 360 MPa), and  $\Phi 8@200$  mm centres as transverse reinforcement (yield strength of 240 MPa). The modulus of elasticity for steel reinforcement was considered as  $E_s=210$  GPa. The ratio for main reinforcement ( $\rho$ ) according to the different codes of practice [8, 9, 11] is equal to the main tension reinforcement area divided by ( $b \times d$ ), this value was set to  $\rho=1.7$  for all beams. Where  $b$  is the breadth of the web and  $d$  is the effective depth of the beam. It is worth mentioning that the behaviour of the studied beams, having such  $a/d$  ratio and adequate reinforcement according to the codes of practice, is in a transition zone between strut-and-tie action and flexural behaviour.

### *Material details*

Two concrete mixes were used with nominal 28-day cube compressive strengths of 35 and 70 MPa. Table 2 indicates details of the mix designs. Mix A for NSC included only cement,

coarse and fine aggregates, and water while Mix B for HSC was designed by using a very low water/cement ratio in conjunction with a high-range water reducing agent to maintain workability besides adding silica fume to the mix to increase strength. Mixing was performed using a rotating mixer. Beams were demolded after 24 hrs of casting, covered with wet burlap, and stored under laboratory conditions for 28 days. In addition, three 150 mm cubes were cast from each batch and tested for compressive strength after a water-curing period of 28 days. Concrete modulus of elasticity were calculated based on the concrete compressive strength for the studied beams according to Eurocode 2 [8] and the values are reported in Table 1.

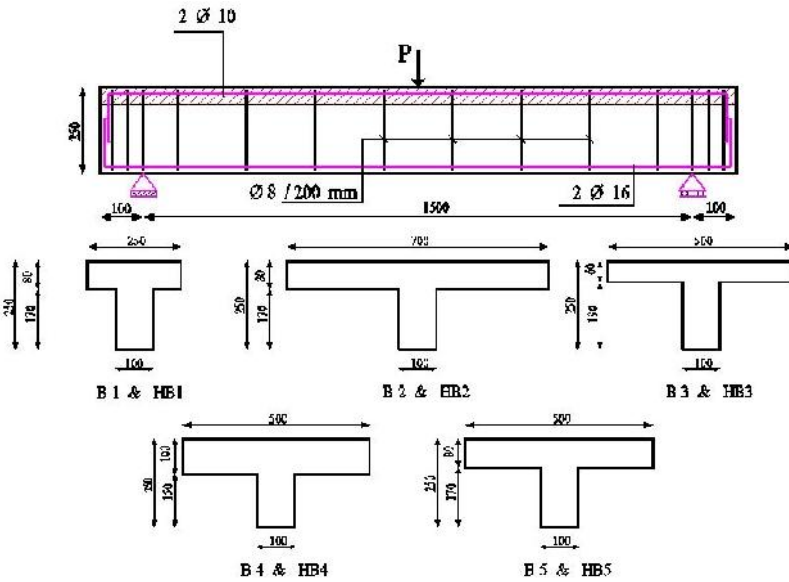


Fig. 1. Specimen details, flange dimensions, and loading position of the tested beams. T

Table 1. Details of the test beam specimens

Concrete Mix	Specimen Notation	Compressive Strength (MPa)	Concrete Modulus of Elasticity ( $E_c$ ) (GPa)	Flange Dimensions	
				Width (B) (mm)	Thickness ( $t_f$ ) (mm)
Mix A (NSC)	B1	35.0	34.0	250	80
	B2	34.5	33.9	700	80
	B3	35.5	34.1	500	60
	B4	36.5	34.3	500	100
	B5	35.0	34.0	500	80
Mix B (HSC)	HB1	72.5	41.1	250	80
	HB2	73.0	41.2	700	80
	HB3	72.0	41.1	500	60
	HB4	71.0	41.0	500	100
	HB5	70.0	41.0	500	80

**Table 2.** Mix constituents and proportions for NSC and HSC

Target Strength (MPa)	Proportions					
	Cement (kg/m <sup>3</sup> )	Sand (kg/m <sup>3</sup> )	*Coarse Aggregate (kg/m <sup>3</sup> )	Silica Fume (kg/m <sup>3</sup> )	Water (kg/m <sup>3</sup> )	Superplasticizer (L/m <sup>3</sup> )
Mix A (35)	350	620	965	--	172	--
Mix B (70)	550	550	1100	75	152	23

\*Natural aggregates of maximum size 10 mm for NSC (Mix A) and Crushed basalt is used for HSC (Mix B)

### ***Loading and instrumentation details***

The tests were carried out in a 500-ton universal testing machine. Each beam was tested as a simply supported beam under one vertical concentrated load (as shown in Fig. 1) by using a vertical hydraulic jack. Deflections were measured using dial gauges (0.01 mm divisions) attached to the test beams bottom surface. All beams were tested during service loads and the dial gauges were removed at a maximum load just prior to failure in a single load cycle. Detection and marking of cracks for each incremental load was made when the load reached its steady state.

## **Results and Discussions**

### ***Cracking loads and load-deflection results***

#### *General observations*

Fig. 2 shows the cracking patterns of the tested T-beams and their load deflection curves are shown in Fig. 3 and Fig.4 for the NSC and HSC, respectively. It is worth mentioning that the maximum loads plotted in the figures for different tested beams are not the failure loads since the dial gauges were removed just prior to failure. It can be seen from the figures that the flange dimensions (breadth and thickness) affected the cracks initiation, propagation, and deflection at different applied loads prior to failure at different degrees. The first crack occurred in the flexural region perpendicular to the direction of the maximum principal tensile stress induced by pure moment. It was noticed that at a higher load level, the shear stresses increased which induced inclined cracks. The cracks propagated, spread and widened due to the effect of combined shear and bending stresses. However, the shear stresses contribution for  $a/d=3.33$  is not as significant as the bending stresses. With increasing the load level to the maximum before removing the dial gauges and stopping the test, both flexural cracks and the diagonal shear ones spread into small cracks in the lower third of the beam web.

#### *NSC beams*

The first crack load values for beams B1, B2, B3, B4, B5 were recorded at 22, 26, 20, 35, 22 kN, respectively. Before stopping the test, shear cracks started from the support and propagated diagonally towards the point load as shown in Fig. 3. However, the failure modes were a flexural failure. The cracks continued growing from the web to the bottom of the flange in the region of the flexural cracks and the flange crushed prior to failure. Increasing the flange thickness to 100 mm for B4, resulted in a higher first crack load while the failure mode did not change. Load-deflection curves presented in Fig. 3 revealed that increasing the flange breadth from 250 mm for B1 to 700 mm for B2 resulted in an increase in the maximum load and corresponding deflection by 22% and 43%, respectively. At a service load level of 80kN, increasing the flange width to 700 mm for B2 resulted in a similar deflection as increasing the flange thickness to 100 mm which was less than that of B1 by 31%. Despite that increasing the

flange thickness from 60 mm, for B3 to 80 mm, for B5 did not affect the maximum applied load which was 90 kN but the corresponding deflection decreased by 15%. A further increase of the flange thickness from 80 mm, for B5 to 100 mm, for B4 resulted in an increase in the maximum load of this beam by 11% and a slight increase in the corresponding deflection by 4%. It can be observed that increasing the flange breadth and thickness led to an increase of the maximum applied load prior to failure and improvement of overall deflection, which is an indication of stiffness increasing.

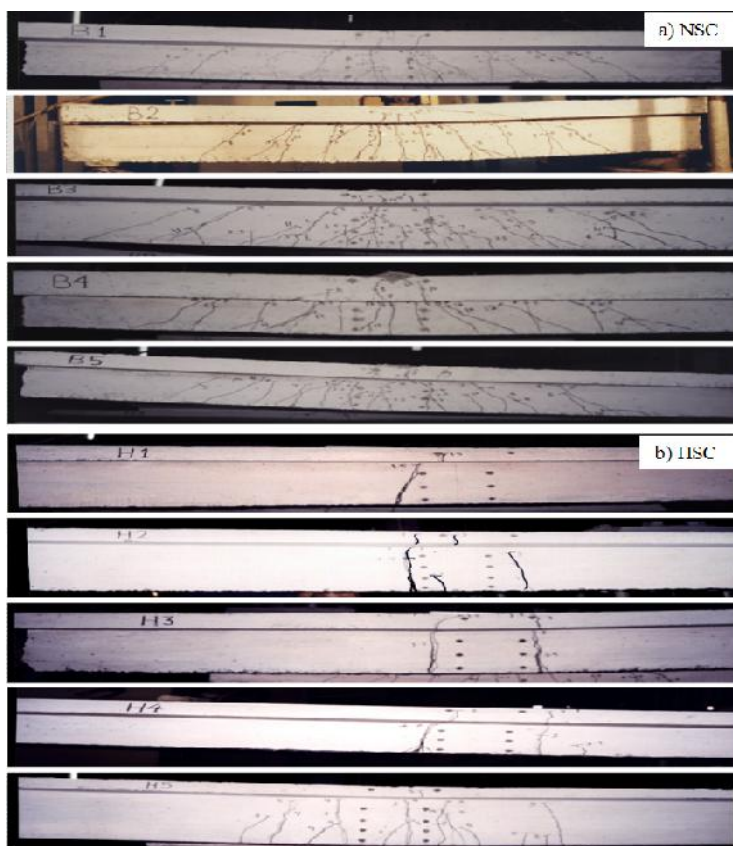


Fig. 2. Crack pattern for a) NSC and b) HSC T-beams prior to failure.

#### *HSC beams*

It was observed from Fig. 4 that the first crack load values for beams HB1, HB2, HB3, HB4, and HB5 were recorded at 20, 15, 30, 33, 20 kN, respectively. It was observed earlier for NSC and now for HSC that the highest first crack load was for the beam which have the maximum flange thickness (B4 and HB4). Fig. 4 shows that increasing flange breadth resulted in concentration of the cracks in the flexural region. Fig. 4 shows also that the maximum applied load prior to failure, for HB2 was almost the same as that of HB4 while the corresponding deflection of HB4 was lower than that of HB2 by 11%. At a service load level of 80kN, the deflection of B4 was less than that of B1 by 23%. In addition, increasing the flange breadth from 250 mm for HB1 to 700 mm for HB2 resulted in an increase in the maximum applied load prior to failure and corresponding deflection by 21% and 55%, respectively.

Moreover, increasing the flange thickness from 60 mm, for HB3, to 80 mm, for HB5, resulted in an increase in the maximum load and corresponding deflection by only 10% and 4%, respectively. A further increase of the flange thickness from 80 mm, for HB5 to 100 mm, for HB4 resulted in a slight increase in the maximum load of this beam by 6% and an increase in the corresponding deflection by 21%. Such effect of flange dimensions may be attributed to the fact that the concrete adheres to reinforcement bars contributes to the overall stiffness, which is mainly dependent on the flange dimensions due to the bond with reinforcement.

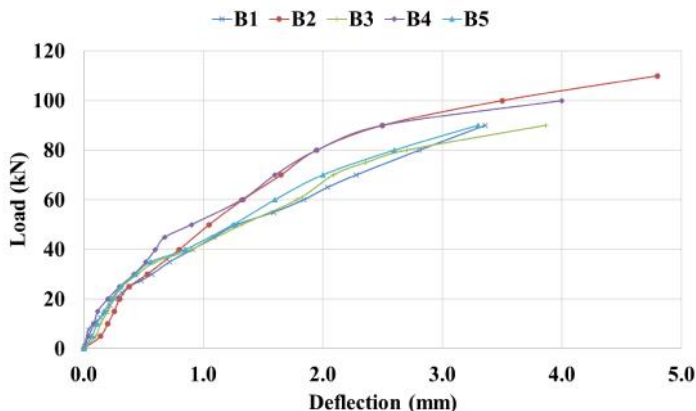


Fig. 3. Load-deflection curves for NSC T-beams prior to failure.

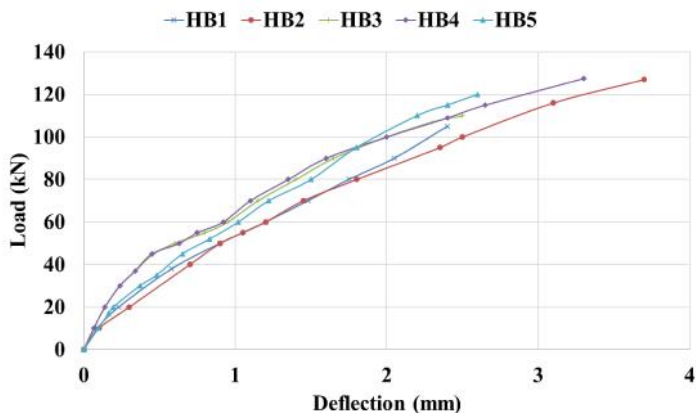


Fig. 4. Load-deflection curves for HSC T-beams prior to failure.

**Equations for predicting the effective moment of inertia for a RC section**

*The available equations in literature*

The effective moment of inertia presented earlier by Branson [7] and adopted in the ACI Building Code [9] by the following equation:

$$I_e = (M_{cr(exp)} / M_a)^3 I_g + [1 - (M_{cr(exp)} / M_a)^3] I_{cr} \leq I_g \tag{1}$$

Where  $I_g$  is the gross moment of inertia for the uncracked section,  $I_{cr}$  is the moment of inertia of the cracked transformed section,  $M_{cr(exp)}$  is the first cracking moment of the beam

calculated from the experimental first crack load at the load-deflection curve, and  $M_a$  is the maximum service load moment acting on the beam.

Al-Zaid et al. [12], modified Branson's effective moment of inertia model to account for non-uniform load configurations accurately after the concrete section cracks [1, 2]. The general form of Eq. (1) is as follows:

$$I_e = (M_{cr(exp)} / M_a)^m I_g + [1 - (M_{cr(exp)} / M_a)^m] I_{cr} \leq I_g \quad (2)$$

Where  $m$  is the experimentally determined exponent for use in the equation. The value of "m" is determined using the following formula,

$$m = \log [(I_{exp} - I_{cr}) / (I_g - I_{cr})] / \log (M_{cr(exp)} / M_a) \quad (3)$$

Where  $I_{exp}$  is the experimental moment of inertia. In a more recent research, Al-Shaikh and Al-Zaid [13] executed an experimental program to study the effect that reinforcement ratio  $\rho$  plays on a reinforced concrete member's effective moment of inertia. The experimental program was conducted by applying a mid-span concentrated load to reinforced concrete beams, of rectangular cross-section, containing varying amounts of reinforcement. The study revealed that Branson's model underestimated the effective moment of inertia of all test specimens. The underestimation of  $I_e$  was approximately 30% in the case of a heavily reinforced member and 12% for a lightly reinforced specimen. The experimental values of the moment of inertia  $I_{exp}$  were calculated from the measured deflections using the elastic deflection formula:

$$I_{exp} = k M L^2 / E_c \Delta_{exp} \quad (4)$$

Where  $L$  is the beam span,  $k$  is a constant depends on the type of loading and end conditions ( $k = 1/12$  for simply supported beams under a mid-span concentrated load),  $\Delta_{exp}$  is the maximum deflection, and  $E_c$  is the elastic modulus of concrete. Al-Shaikh and Al-Zaid [13] incorporated the effect of  $\rho$  in the exponent "m" by curve fitting in order to keep the form of Eq. (4) with minimum modification as follows:

$$m = 3.0 - 0.8\rho \quad (5)$$

Wickline [19] carried out a similar operation in which three values for "m" was proposed depending on the reinforcement ratio. Ghali et al. [20] proposed another relationship for predicting effective moment of inertia as follows:

$$I_e = I_g I_{cr} / [I_{cr} + (1 - 0.5(M_{cr} / M_a)^2)(I_g - I_{cr})] \quad (6)$$

Bischoff [14] in his critical evaluation of equations commonly used to compute short-term deflection for steel and fibre reinforced polymer (FRP) reinforced concrete beams, developed an alternative expression for calculating beam deflection with a rational approach that incorporates a tension-stiffening model adopted in Europe. Later, Bischoff and Scanlon [21] used the developed equation given below. Where;  $\eta = 1 - I_{cr} / I_g$  and  $\gamma = 1$  for single monotonic loading.

$$I_e = I_{cr} / [1 - \gamma\eta (M_{cr}/M_d)^2] \tag{7}$$

The previous equations were applied to the experimental results of this study. It was noticed from Fig. 5 that for NSC beams, the predicted value by (7) was almost similar to that predicted by (1). The average values for the ratios of  $I_{exp}/I_e$  ranged from 1.16 to 1.55 and the standard deviations ranged from 0.16 to 0.32. For HSC T-beams, the prediction using previous equations was not accurate. The average values for the ratios of  $I_{exp}/I_e$  ranged from 1.5 to 1.94 and the standard deviations ranged from 0.13-0.38.

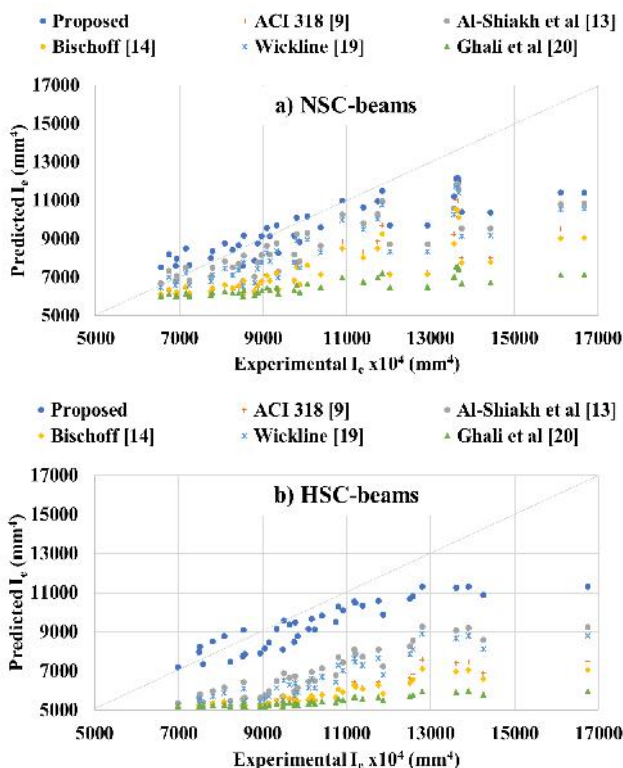


Fig. 5. Comparison between different equations for predicting  $I_e$  values of a) NSC and b) HSC T-beams.

*Proposed Equations*

Al-Shaikh and Al-Zaid [13] equation was in a better agreement with the experimental results of NSC T-beams compared to the other equations. The main variable in their work was the effect of “m” values. However, the “m” value obtained by Al-Shaikh and Al-Zaid [13] is a function of tensile reinforcement ratio ( $\rho$ ). Therefore, the authors attempted to modify “m” value in Eq. (5) by changing the factor multiplied by the tensile reinforcement ratio ( $\rho$ ) using curve fitting in order to account for the differences between rectangular, T-beams for NSC and HSC beams. Eq. (8) is applicable to the NSC T-beams and (9) is applicable to the HSC T-beams:

$$m = 3 - 1.1 \rho \tag{8}$$

$$m = 3 - 1.4 \rho \tag{9}$$

The proposed “*m*” values obtained from Eq. (8) and Eq. (9) were then used along with Eq. (2) to Eq. (4) to calculate the  $I_e$  for the studied beams, and the experimental versus predicted values plots are shown in Fig. 5. For NSC, it was noticed that the proposed equation gave a better average value of 1.06 than that of the other equations from the literature and the standard deviation was 0.16. For HSC, the proposed equation gave a higher accuracy of predictions compared with the results obtained by applying the other equations with an average value of 1.12 and the standard deviation was 0.15. This may be attributed to the fact that the effect of tensile reinforcement is more significant for HSC than that for NSC. The proposed equations show a better agreement with the experimental work than that of the others from literature, especially for HSC T-beams. Fig. 5 shows that most of the predicted values by using the other equations from the literature were underestimating the predicted values of  $I_e$ , to different degrees.

A pilot study was carried out in order to test the proposed equations against the available experimental work of other researchers cited in the literature. Table 3 shows the prediction of effective moment of inertia of some T-beams experimentally tested by five researchers in literature. The results obtained by the proposed equations are in good agreement with the experimental work. However, more work is needed in the future to calibrate these equations by verification with more accurate methods such as finite element techniques or prediction of more experimentally tested NSC and HSC T-beams.

**Table 3.** Validation of the proposed equations for T-beams from other studies

Reference/ Concrete type	Designation	$f_c'$ (MPa)	$\rho$ (%)	$E_c$ (GPa)	$I_{exp}/I_{e(Prop.)}$
[22] NSC	B1	35.0	2.0	27.80	1.09
	B2	35.0	2.0	27.80	1.02
	B3	35.0	2.0	27.80	1.21
	B4	35.0	2.0	27.80	0.89
	B5	35.0	2.0	27.80	0.95
[23] NSC	BRM25	22.05	0.65	23.82	1.07
	BN25	24.20	0.65	23.27	1.03
	BSK25	24.90	0.65	23.38	1.08
	BSK50	39.25	0.65	30.57	1.11
	BN50	36.43	0.65	27.13	1.03
[24] HSC	BRM50	37.53	0.65	27.69	1.02
	B36L-1	54.47	2.90	29.650	1.19
	B36L-2	54.74	2.90	31.000	1.05
	B44-1	58.40	2.40	30.700	1.12
	B44-2	58.88	2.40	30.700	1.10
[25] HSC	B22-1	80.87	1.30	35.800	0.81
	B22-2	75.83	1.30	33.400	0.61
	A211	42.80	1.25	30.748	0.98
	B211	73.60	1.25	40.321	0.78
	D211	114.50	1.90	50.292	0.77
[26] HSC	E211	126.20	1.90	52.799	0.84
	B1	96.0	2.50	41.188	1.22
	B2	100.0	2.50	42.038	0.92
	B3	85.0	2.50	38.575	1.51
	B4	90.0	2.50	39.880	1.61
	B5	87.0	2.50	39.210	1.06
	B6	88.0	2.50	39.435	1.33

## Conclusions

Based on the experimental results reported in the paper, the following conclusions are drawn:

- Increasing the flange dimensions (breadth and thickness) delayed the cracks initiation, propagation, increased the maximum applied load prior to failure, and reduced the short term deflection of the studied beams to different degrees.
- Using of the currently available equations in literature for predicting the  $I_e$  will underestimate experimental values to a high degree especially for the HSC T-beams.
- The proposed equations for the NSC and HSC T-beams show a good accuracy and repeatability in predicting the  $I_e$  values with an average ratio of  $I_{exp}/I_e=1.06$  and 1.12, the standard deviation of 0.16 and 0.15 for the NSC and HSC T-beams, respectively..
- The pilot study shows that the results obtained by the proposed equations are promising. However, more work is needed to calibrate these equations by verification with more accurate methods such as finite element technique or predicting the results of more experimentally tested NSC and HSC T-beams in the literature.

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