Joint Computation and Communication Design for UAV-Assisted Mobile Edge Computing in IoT

Tiankui Zhang, Yu Xu, Jonathan Loo, Dingcheng Yang, Lin Xiao

Abstract—Unmanned aerial vehicle (UAV)-assisted mobile edge computing (MEC) system is a prominent concept, where a UAV equipped with a MEC server is deployed to serve a number of terminal devices (TDs) of Internet of Things (IoT) in a finite period. In this paper, each TD has a certain latency-critical computation task in each time slot to complete. Three computation strategies can be available to each TD. First, each TD can operate local computing by itself. Second, each TD can partially offload task bits to the UAV for computing. Third, each TD can choose to offload task bits to access point (AP) via UAV relaying. We propose a new optimization problem formulation that aims to minimize the total energy consumption including communication-related energy, computation-related energy and UAV’s flight energy by optimizing the bits allocation, time slot scheduling and power allocation as well as UAV trajectory design. As the formulated problem is non-convex and difficult to find the optimal solution, we solve the problem by two parts, and obtain the near optimal solution with within a dozen of iterations. Finally, numerical results are given to validate the proposed algorithm, which is verified to be efficient and superior to the other benchmark cases.

Index Terms—Internet of Things, mobile edge computing, resource allocation, trajectory optimization, UAV communication.

I. INTRODUCTION

Recently, with the advancement in Internet of Things (IoT) technology, various up-to-date applications, e.g., the augmented reality (AR), virtual reality (VR), autonomous driving and agriculture monitoring, are changing our experience. Some terminal devices (TDs) related to the Internet of Things (IoT) such as smart phones, monitoring sensors and wearable devices spring up in our life [1] [2]. However, the computation demands for IoT devices are also becoming higher while the computing capacity of these devices is limited. Mobile edge computing (MEC) is considered as a new technology to overcome the limitations by providing cloud-like computing. By deploying computing resource in close proximity to IoT devices (i.e., locating MEC servers at a wireless access point (AP) or base station), it can efficiently reduce the delay and save the computation resource at these devices by the way of computation task offloading [3] [4]. Therefore, MEC has the potential to provide the service of solving the computation-intensive and latency-critical tasks for devices. In general, the MEC server deployment is fixed, which means that it can not exploit its mobility to move closer to TDs, by which the latency or energy consumption of the devices would be further reduced.

Due to the high flexible mobility, unmanned aerial vehicle (UAV) has attracted significant research interest in academia [5]-[9]. In wireless communications, UAV has been applied in various scenarios, such as nonorthogonal multiple access (NOMA) networks [10], mmWave communications [11] and caching [12] [13]. Also, the three-dimensional coverage performance for cellular network-connected UAVs that act as aerial users is also investigated in [14]. In addition, UAV relaying [15]-[17] is also an important application that can efficiently expand the communication coverage. By utilizing UAV as a relay, two users with communication channel blocked can be linked. This gives a new method to help local resource-limited users access to the remote resources.

The new setup by utilizing UAV to assist computing in MEC systems poses new opportunities to solve the challenges in communication and computation design, and several prior related works have been done for this [18]- [28]. Specifically, the work [18] considers that a UAV is deployed to provide computation service for TDs, and a minimization problem of sum of the maximum delay among users is proposed by optimizing the offloading ratio, users scheduling and UAV trajectory. In work [19], the computation rate maximization problem in a UAV-assisted MEC is investigated. The authors in [20] focus on minimizing the average weighted energy consumption of TDs, and the optimal solution is obtained by decomposing the primal problem into three subproblems. The authors in [21] investigate computation energy consumption of mobile terminal minimization problem, but the UAV trajectory is not optimized. Hua et al. [23] consider a UAV to help TD offload bits, the TD can compute locally as well as can offload bits to the UAV. Besides, the works [24] and [25] study the UAV energy minimization problem and task completion time minimization problem in cellular-connected UAV MEC systems, respectively. Bai et al. [26] focus on the security in UAV-assisted MEC systems, where a potential passive eavesdropper can capture the offloading bits from the UAV to AP via eavesdropping channel. Also, Du et al. [27] study the energy efficiency of the UAV in a MEC system, by minimizing the hovering energy and computation energy of the UAV. In addition, the work [28] study the problem described as the offloading bits from users to UAV maximization, subject to each user’s quality of service (QoS). These existing works related to UAV-assisted MEC systems mainly focus on the computing bits offloading only between UAV(s) and users.

Different from the existing works, we propose a framework in which the UAV acts as a relay to assist bits offloading for TDs. Specifically, the UAV can not only provide the computation service but also can provide the communication service for TDs by forwarding the received bits to AP for remote computation. Thus, our proposed framework further enhances the computing ability of the MEC systems.

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The UAV-assisted MEC systems in IoT are studied in this paper, in which the UAV is considered as a helper that not only helps computing the bits offloaded from TDs but also acts as a decode-and-forward (DF) relay to assist task bits transmit from TDs to AP. Considering the practical terrible channel environment between the TDs and remote AP, and in order to clearly shed light on the essence of our proposed system, it is assumed that the direct communication links between TDs and AP are blocked. Also, the total energy on UAV is enough to support propulsion and complete the task during the period. These TDs need to process their collected data, such as the video file, temperature information and movement data, they need to transmit a part of task bits to the UAV for processing if they are unable to compute locally. For a given period, each TD needs to complete the required latency-critical task in per time slot. In addition, considering the AP is located on the ground, it can be equipped with a or several powerful MEC server(s). Thus, the maximum computing rate at the AP would be much larger than the bits offloading rate from the UAV in our setup. Therefore, it is reasonable to assume that the computing time at AP in each time slot is negligible. Our goal is to minimize the sum energy of communication-related energy, computation-related energy and UAV’s flight energy subject to the constraints on communication and computation resource allocation, computation causality constraint and UAV trajectory design. In our design, the UAV’s mobility is restricted by the maximum speed and initial/final location, and it serves the TDs in an orthogonal frequency-division multiple access (OFDMA) manner. In summary, the main contributions of this work are presented as follows.

- We propose a new framework of UAV-assisted MEC system in IoT. Our proposed framework fills the gap that jointly considers the task offloading strategy and UAV relay communication in MEC systems, which provides useful insights and guidelines for designing the similar problems in practice. In our design, the required computation bits can be computed by TDs locally, or offloaded to the UAV for computing. Besides, the required task bits also can be transmitted to the AP for computing via UAV relaying. This mode can further expand the computation resource scale and provide a new opportunity to solve the challenges in traditional MEC systems.

- We formulate a total energy consumption minimization problem, by optimizing the computation bits allocation, time slot scheduling, transmit power allocation and UAV trajectory. A problem decomposition method is adopted to tackle the non-convex problem in two parts that are solved by the Lagrangian duality method and successive convex approximation (SCA) technique, respectively.

- We present the numerical results that show the superiorities of our proposed design, as compared with other benchmark designs. On the one hand, the proposed algorithm can be guaranteed to converge within a dozen of iterations. On the other hand, the total energy consumption obtained by the proposed algorithm is always lowest, indicating the significant effectiveness of our design.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a UAV-assisted MEC system in IoT as shown in Fig. 1, where a UAV is deployed as a mobile DF relaying over an area of interest. The UAV is dispatched to help computation bits of TDs that are denoted by a set $K = \{1, 2, \ldots, K\}$ transmit to AP equipped with MEC functionality for computing. For convenience, we use the notation $u_k$ to denote TD $k$ in this paper. Meanwhile, the UAV is also equipped with a MEC server to provide computation operation for TDs. The UAV, each TD and AP are assumed to be equipped with one single antenna, respectively. Without loss any generality, we assume that the UAV flies from an initial location $q_0$ to final location $q_F$. The flight altitude is fixed at $H$ that effectively avoids any collisions.

The period time for the UAV flight is expressed by $T$. Considering a 3D Cartesian coordinate system, the UAV’s location projected on the horizontal plane in any time instant $t \in [0, T]$ can be represented by $q(t) = (x(t), y(t))$. In addition, the locations of AP and each TD $k \in K$ are fixed at $w_a = (x_a, y_a)$ and $w_k = (x_k, y_k)$, respectively. For convenience, we use sufficiently small constant $\delta$ to divide the period $T$ into $N$ slots with equal size, which are expressed by a set $\mathcal{N} = \{1, 2, \ldots, N\}$. In each time slot, the UAV can be considered to be static. Thus, the UAV’s location in any time slot $n \in \mathcal{N}$ can be denoted by $q[n] = [x[n], y[n]]^T$, with $q(t) = q(\delta t, n) = q[n]$. Hence, the distance between the TD $k$ and UAV/helper in each time slot $n \in \mathcal{N}$ can be denoted by $d_{a,k}[n] = \sqrt{H^2 + \|q[n] - w_k\|^2}$, where $\| \cdot \|$ denotes Euclidean norm. Similarly, the distance between the UAV and AP in each time slot can be denoted by $d_{ba}[n] = \sqrt{H^2 + \|q[n] - w_a\|^2}$.

For each TD $k \in K$, it has a latency-critical computation task requirement in each time slot $n \in \mathcal{N}$, i.e., each user needs to complete at least $L^{\min}_{k,n}$ bits of computation task in each time slot $n$. Considering its limited computing ability, each TD can offload the computation bits to the UAV via wireless transmit for either computing or relaying. Let $l_{u,k}[n]$, $l_{h,k}[n]$ and $l_{a,k}[n]$ denote the amount of computation bits allocated for local computing, offloading to UAV for computing and offloading to AP for computing via relaying (or offloading to UAV for relaying) in each time slot, respectively. Thus we have,

\begin{align}
    l_{u,k}[n] &\geq 0, l_{h,k}[n] \geq 0, l_{a,k}[n] \geq 0, \\
    l_{u,k}[n] + l_{h,k}[n] + l_{a,k}[n] &\geq L^{\min}_{k,n}, \forall k, n. 
\end{align}

A. Communication Model

Note that the delay and energy consumption for results sending back from UAV to TDs and that from AP to UAV are omitted since the size of results is much smaller than offloaded data size [19] [25]. As shown in Fig. 2, we consider a computation bits offloading protocol of each TD. Specifically, in each time slot $n \in \mathcal{N}$, the TDs can offload their tasks to the UAV for computing. It is assumed that the wireless channel between the UAV and TD $k$ is dominated by LoS link [19]
carriers with size of bits forwarding to AP from UA V, respectively. The size of each subslots that are dedicated to be allocated for bits offloading \(k\) power dense at the UA V. Assume that the transmit power of TD for computing is denoted as \(\beta\) loss model. Thus, the channel power gain from TD to AP via UAV. Thus, the channel power gain from the UAV to AP is obtained as

\[
h_{uk}[n] = \beta_{uk}d_{uk}^{-2}[n] = \frac{\beta_{0}}{H^2 + ||q[n] - w_k||^2},
\]

where \(\beta_{0}\) denotes the channel gain at the reference distance \(d_0 = 1\) meter. Besides, the TDs also can offload their tasks to AP.

In Fig. 2, for each TD \(k\), each time slot \(n \in N\) is divided into 3 subslots that are dedicated to be allocated for bits offloading to UAV for computing, bits offloading to UAV for relaying and bits forwarding to AP from UAV, respectively. The size of each subslot is determined by variable \(\tau_{k,m}[n]\), with \(m = \{1, 2, 3\}\), which satisfies the following constraints,

\[
\sum_{m=1}^{3} \tau_{k,m}[n] \leq 1, \forall k, n,
\]

\[
0 \leq \tau_{k,m}[n] \leq 1, \forall k, n, m = \{1, 2, 3\}.
\]

It is assumed that an OFDMA is applied in the system. The total available bandwidth \(B\) is equally divided into \(K\) subcarriers with size of \(B_{0} = \frac{B}{K}\) for each TD. The transmit power of TD for offloading bits to UAV for computing in each time slot \(n\) is denoted by \(p_{k,1}[n]\). Therefore, the achievable offloading rate in bits-per-second (bps) from the TD to UAV for computing is denoted as

\[
r_{uh}(p_{k,1}[n], q[n]) = B_{0} \log_{2} \left(1 + \frac{p_{k,1}[n]h_{uk,h}[n]}{N_0B_{0}}\right)
\]

\[
= B_{0} \log_{2} \left(1 + \frac{\left[p_{k,1}[n]h_{0,k}[n]\right]}{N_0B_{0}g_{0}d_{0}^{2}}\right), \forall k, n,
\]

where \(g_{0}\) denotes the reference received signal-to-noise ratio (SNR) at UAV for \(d_0 = 1\) meter, and \(N_0\) denotes noise power dense at the UAV. Assume that the transmit power of TD to UAV for relaying in the second subslot with duration of \(\delta_{1}\tau_{k,2}[n]\) is denoted by \(p_{k,2}[n]\). Thus, the achievable offloading rate in bps from the TD to UAV for relaying is given as

\[
r_{rh}(p_{k,2}[n], q[n]) = B_{0} \log_{2} \left(1 + \frac{p_{k,2}[n]h_{0,k,h}[n]}{N_0B_{0}}\right)
\]

\[
= B_{0} \log_{2} \left(1 + \frac{\left[p_{k,2}[n]h_{0,k}[n]\right]}{N_0B_{0}g_{0}d_{0}^{2}}\right), \forall k, n.
\]

Similarly, the achievable forwarding rate from the UAV to AP in bps is given as

\[
r_{hf}(p_{k,3}[n], q[n]) = B_{0} \log_{2} \left(1 + \frac{p_{k,3}[n]h_{k,a}[n]}{N_1B_{0}}\right)
\]

\[
= B_{0} \log_{2} \left(1 + \frac{\left[p_{k,3}[n]h_{0,k,a}[n]\right]}{N_1B_{0}g_{0}d_{0}^{2}}\right), \forall k, n,
\]

where \(g_0 = \frac{\beta_0}{N_1B_0}\) denotes the reference received SNR at AP

for \(d_0 = 1\) meter, and \(N_1\) denotes noise power dense at AP.

In addition, we assume that the UAV is able to store the unprocessed offloading bits from TDs in its memory if the offloading rate exceeds its computing ability. Consequently, we can obtain the following computation causality condition,

\[
\sum_{i=1}^{N} l_{k,i}[n] \leq \sum_{i=1}^{K} \delta_{i} \tau_{k,1}[n] r_{uh}(p_{k,1}[n], q[n]), \forall k, n,
\]

Assuming that the processing delay at the DF relay is one subslot, the computing bits \(l_{k}[n]\) should satisfy the expression shown in (11).

In this model, the total communication-related energy consumption is considered, given by

\[
E_{comm} = \delta_{1} \sum_{m=1}^{N} \sum_{k=1}^{K} (\tau_{k,m}[n]p_{k,m}[n])
\]

\[\text{(12)}\]

B. Computation Model

Let \(\tau_{k} > 0\) denotes the required CPU cycles for computing each one bit at the user \(k\), and \(\kappa_{k} > 0\) represents the effective capacitance coefficient effected by chip architecture at TD \(k\). It is assumed that all TDs have same CPU cycles and capacitance coefficient, i.e., \(c_{u} = c_{u}\), \(\kappa_{u} = \kappa_{u}, \forall k\). In order to help TDs complete computation tasks in each time slot, as shown in (2), we assume that the CPU cycles and capacitance coefficient of the MEC server at the UAV are \(c_{h} > 0\) and \(\kappa_{h} > 0\), respectively. In addition, the computation capacity of the AP is assumed to be sufficiently powerful so that the computing time at the AP can be negligible in our setup. The maximum CPU frequency at each TD and UAV is denoted by \(f_{u,max}\) and \(f_{h,max}\), respectively. As a result, in any time slot, we have

\[
c_{u} I_{u,k}[n] \leq \delta_{i} f_{u,max}\), \forall k, n,
\]

\[
c_{h} I_{h,k}[n] \leq \delta_{h} f_{h,max}, \forall k, n,
\]

where \(f_{h,max} = \frac{\kappa_{h} c_{h}}{K}\), indicates that the total frequency of the UAV is equally divided into \(K\) parts that are allocated to each TD, respectively. Based on [25], the energy consumption in each time slot for local computing is expressed as

\[
E_{comp}^{u,k}[n] = \frac{c_{u}(c_{u} h_{u,k}[n])^{2}}{\delta_{u}^{2}}, \forall k, n.
\]

Similarly, the energy consumption in each time slot for UAV computing is expressed as

\[
E_{comp}^{h}[n] = \sum_{k=1}^{K} \left(\frac{\kappa_{h} c_{h}}{\delta_{h}^{2}}(c_{h} I_{h,k}[n])^{2}\right), \forall n.
\]

it is worth mentioning that in the first time slot of \(n = 1\), the available time duration for UAV computing is \((\delta_{1} - \delta_{1} \tau_{k,1}[1])\) s. However, considering the time slot size \(\delta_{1}\) in our design is chosen to be quite small so that we have \(\delta_{1} \tau_{k,1}[1] \ll T\). Thus, the computing time for the UAV in first time slot \(n = 1\) can be approximated to be \(\delta_{1}\). As a result, the total computation-related energy consumption can be denoted as

\[
E_{comp} = \sum_{n=1}^{N} \sum_{k=1}^{K} E_{comp}^{u,k}[n] + \sum_{n=1}^{N} E_{comp}^{h}[n].
\]

C. UAV Mobility and Flight Energy Consumption Model

In the proposed system, an altitude-fixed rotary-wing UAV is considered. In practice, this UAV flies from an initial location to a final location, during which its speed is constrained by a maximum speed \(V_{max}\). Hence, we have

\[
q[1] = q_{0},
\]

\[\text{(18a)}\]
\[ l_{a,k}[n] \leq \min(\delta_i \tau_{k,2}[n] r_{uh} (p_{k,2}[n], q[n]), \delta_i \tau_{k,3}[n] r_{ha} (p_{k,3}[n], q[n])), \forall k, n, \]  
\[ (11) \]

\[ q[N + 1] = q_F, \quad ||q[n] + 1 - q[n]||^2 \leq (\delta_i V_{\text{max}})^2, \forall n. \quad (18b) \]

Based on [31], the power consumption of flight for rotary-wing UAV is modeled as

\[ P_0 \left( 1 + \frac{3 ||v[n]||^2}{U_{\text{tip}}^2} \right) + P_t \left( \sqrt{1 + \frac{||v[n]||^4}{4v_0^4} - \frac{||v[n]||^2}{2v_0^2}} \right)^{\frac{1}{2}} + \frac{1}{2} d_{\text{ps,Al}} ||v[n]||^3, \forall n \in \mathcal{N}. \quad (19) \]

where \( P_0 \) and \( P_t \) represent the blade profile power and induced power in hovering status, respectively. The other parameters of \( U_{\text{tip}}, v_0, d_0, p, s \) and \( A \) related to the UAV’s aerodynamics are given in Table I in Section IV based on the work [31]. To achieve the UAV’s speed \( v[n] \), we have

\[ v[n] = \frac{q[n + 1] - q[n]}{\delta_i}, \forall n \in \mathcal{N}. \quad (20) \]

Thus, the UAV’s flight energy consumption during the period time is expressed as

\[ E_{fiy} = \delta_i \sum_{n=1}^{N} P_0 \left( 1 + \frac{3 ||v[n]||^2}{U_{\text{tip}}^2} \right) + P_t \left( \sqrt{1 + \frac{||v[n]||^4}{4v_0^4} - \frac{||v[n]||^2}{2v_0^2}} \right)^{\frac{1}{2}} + \frac{1}{2} d_{\text{ps,Al}} ||v[n]||^3, \forall n \in \mathcal{N}. \quad (21) \]

D. Problem Formulation

According to the discussion above, we formulate the objective problem as a sum of communication-related energy consumption and computation-related energy consumption minimization, which is subjected to task allocation, time slot scheduling, transmit power allocation and UAV trajectory design. Specifically, the problem is formulated as

\[
\begin{align*}
\text{(P1):} \quad & \min \quad E_{\text{comm}} + E_{\text{comp}} + \mu E_{fiy} \\
\text{s.t.} \quad & \begin{align*}
0 \leq p_{k,1}[n] & \leq P_{u,max}, \forall k, n, \\
0 \leq p_{k,2}[n] & \leq P_{v,max}, \forall k, n, \\
0 \leq p_{k,3}[n] & \leq P_{h,max}, \forall k, n,
\end{align*} \quad (22a) \quad \text{(22b)} \quad \text{(22c)}
\end{align*}
\]

where \( L = \{l_{u,k}[n], l_{h,k}[n], l_{a,k}[n]\}, \tau = \{\tau_{k,m}[n]\}_{m=1}^{3}, P = \{p_{k,i}[n]\}_{i=1}^{3}, Q = \{q[n], v[n]\}_{n=1}^{N}, \quad P_{u,max} \text{ and } P_{h,max} \text{ stand for the maximum transmit power at each TD and UAV, respectively. Like [19], let } \nu \text{ denote the weight with regard to the UAV’s flight energy consumption to ensure the fairness for TDs.}

Obviously, problem (P1) is a non-convex problem due to the non-convexity in the constraints (10) and (11) as well as in the objective function. To tackle this, the primal problem (P1) is decomposed into two manageable subproblems, which are analyzed in the following sections.

III. ENERGY MINIMIZATION WITH FIXED TRAJECTORY

For any given UAV trajectory \( Q \), and let \( E_{k,m}[n] = t_{k,m}[n] p_{k,m}[n] \) and \( t_{k,m}[n] = \delta_i \tau_{k,m}[n] \), with \( m = \{1, 2, 3\} \). The primal problem (P1) is formulated as problem (P2).

\[
\begin{align*}
\text{(P2):} \quad & \min \quad \sum_{n=1}^{N} \sum_{k=1}^{K} [E_{k,1}[n] + E_{k,2}[n] + E_{k,3}[n] + \frac{\nu_k (c_{u,k}[n])^3}{\delta_i^2} + \frac{\nu_h (c_{h,k}[n])^3}{\delta_i^2}], \\
\text{s.t.} \quad & \begin{align*}
\sum_{n=1}^{N} \sum_{k=1}^{K} & E_{k,1}[n] \leq \sum_{n=1}^{N} \sum_{k=1}^{K} t_{k,1}[n] r_{uh} (E_{k,1}[n], t_{k,1}[n]), \forall k, n, \\
\sum_{n=1}^{N} \sum_{k=1}^{K} & E_{k,2}[n] \leq \sum_{n=1}^{N} \sum_{k=1}^{K} t_{k,2}[n] r_{uh} (E_{k,2}[n], t_{k,2}[n]), \forall k, n, \\
\sum_{n=1}^{N} \sum_{k=1}^{K} & E_{k,3}[n] \leq \sum_{n=1}^{N} \sum_{k=1}^{K} t_{k,3}[n] r_{ha} (E_{k,3}[n], t_{k,3}[n]), \forall k, n, \\
\sum_{n=1}^{N} \sum_{k=1}^{K} & \nu_k (c_{u,k}[n])^3 \leq \sum_{n=1}^{N} \sum_{k=1}^{K} \nu_k t_{k,1}[n], \forall k, n, \\
\sum_{n=1}^{N} \sum_{k=1}^{K} & \nu_h (c_{h,k}[n])^3 \leq \sum_{n=1}^{N} \sum_{k=1}^{K} \nu_h t_{k,2}[n], \forall k, n, \\
\sum_{n=1}^{N} \sum_{k=1}^{K} & \omega_k (c_{h,k}[n])^3 \leq \sum_{n=1}^{N} \sum_{k=1}^{K} \omega_k t_{k,3}[n], \forall k, n,
\end{align*} \quad (23a) \quad \text{(23b)} \quad \text{(23c)} \quad \text{(23d)} \quad \text{(23e)} \quad \text{(23f)} \quad \text{(23g)} \quad \text{(23h)} \quad \text{(23i)} \quad \text{(23j)} \quad \text{(23k)} \quad \text{(23l)} \quad \text{(23m)}
\]

\[
\begin{align*}
\sum_{n=1}^{N} \sum_{k=1}^{K} l_{a,k}[n] & \leq \sum_{n=1}^{N} \sum_{k=1}^{K} t_{k,1}[n] r_{uh} (E_{k,1}[n], t_{k,1}[n]), \forall k, n, \\
\sum_{n=1}^{N} \sum_{k=1}^{K} l_{a,k}[n] & \leq \sum_{n=1}^{N} \sum_{k=1}^{K} t_{k,2}[n] r_{uh} (E_{k,2}[n], t_{k,2}[n]), \forall k, n, \\
\sum_{n=1}^{N} \sum_{k=1}^{K} l_{a,k}[n] & \leq \sum_{n=1}^{N} \sum_{k=1}^{K} t_{k,3}[n] r_{ha} (E_{k,3}[n], t_{k,3}[n]), \forall k, n, \\
\sum_{n=1}^{N} \sum_{k=1}^{K} \omega_k (c_{h,k}[n])^3 & \leq \sum_{n=1}^{N} \sum_{k=1}^{K} \omega_k t_{k,3}[n], \forall k, n,
\end{align*} \quad (24)
\]

Lemma 1: Problem (P2) is a convex problem.

Proof: Firstly, the objective function of problem (P2) is convex with respect to \( E, l_{u,k}[n] \) and \( l_{h,k}[n] \). Then, it can be easy find that the expressions in constraints (23d)-(23l) are linear. \( f(x, t) = t \log(1 + \frac{1}{x}) \) with \( t > 0 \) is concave [32]. Therefore, the expressions \( t_{k,1}[n] r_{uh} (E_{k,1}[n], t_{k,1}[n]) \), \( t_{k,2}[n] r_{uh} (E_{k,2}[n], t_{k,2}[n]) \) and \( t_{k,3}[n] r_{ha} (E_{k,3}[n], t_{k,3}[n]) \) respectively in constraints (23a)-(23c) are concave. Thus, problem (P2) is proved to be convex.

In order to achieve the closed-form solutions and give more insights into the proposed problem (P2), we choose the Lagrange duality method to solve this problem in this paper. By introducing the nonnegative dual variables \( \lambda_{k,n}, \mu_{k,n}, \nu_{k,n}, \omega_{k,n} \) and \( \eta_{k,n} \) that are corresponding to the constraints (23a)-(23d) and (23k), respectively, and let \( \lambda = \{\lambda_{k,n}\}, \mu = \{\mu_{k,n}\}, \nu = \{\nu_{k,n}\}, \omega = \{\omega_{k,n}\} \) and \( \eta = \{\eta_{k,n}\} \), then the Lagrange function of problem (P2) is
In (24), note that $\hat{\lambda}_{k,n}$ is a new defined parameter that satisfies 

$$\hat{\lambda}_{k,n} = \sum_{i=n}^{\infty} \lambda_{k,i}. \text{ Thus, the dual function of problem (P2) can be denoted by } g(\lambda, \mu, \nu, \omega, \eta), \text{ given as }$$

$$g(\lambda, \mu, \nu, \omega, \eta) = \min_{k,t,E} \mathcal{L}(L, t, E, \lambda, \mu, \nu, \omega, \eta)$$

s.t. (23e)-(23j), (23i).

**Lemma 2:** In order to make $g(\lambda, \mu, \nu, \omega, \eta)$ bounded, the expression of $(\mu_{k,n} + \nu_{k,n} - \omega_{k,n}) \geq 0$ must hold.

**Proof:** Lemma 2 can be shown by contradiction. Assume that $(\mu_{k,n} + \nu_{k,n} - \omega_{k,n}) < 0$, thus the value of $l_{a,k}[n]$ would $\rightarrow +\infty$ in order to minimize the objective function. Thus, the value of dual function $g(\lambda, \mu, \nu, \omega, \eta)$ would be minus infinity. This lemma is proved.

As a result, the dual problem of problem (P2) can be written as

$$(D2): \max_{\lambda, \mu, \nu, \omega, \eta} g(\lambda, \mu, \nu, \omega, \eta)$$

s.t. $\lambda \geq 0, \mu \geq 0, \nu \geq 0, \omega \geq 0, \eta \geq 0$  

$$(\mu_{k,n} + \nu_{k,n} - \omega_{k,n}) \geq 0, \forall k, n.$$  

Due to problem (P2) is convex, the Slater’s condition can be satisfied [32] and thus the strong duality holds between (P2) and (D2). So we can obtain the optimal solution of problem (P2) by solving its dual problem, i.e., problem (D2).

A. Obtaining $g(\lambda, \mu, \nu, \omega, \eta)$ by Solving Problem (25)

For any given value of $(\lambda, \mu, \nu, \omega, \eta)$ in the feasible set of problem (D2), the dual function can be obtained by solving problem (25). Note the problem (25) can be decomposed into $KN$ independent subproblems, and each one is further decomposed into several subproblems as follows.

**Lemma 3:** By solving subproblem (L1) with KKT, the optimal solution can be denoted as

$$E_{k,1}[n] = p_{k,1}[n]t_{k,1}[n],$$

$$p_{k,1}[n] = \left[ \frac{\lambda_{k,n}B_0}{\ln 2} - \frac{1}{\gamma_0} \right]^{\frac{1}{\gamma_0}}_{\mu_{k,n}}$$

$$t_{k,1}[n] = \begin{cases} \delta_t, & \text{if } p_{k,1}[n] - \lambda_{k,n}r_{u,k}[n] + \eta_{k,n} < 0 \\ \epsilon(0, \delta_t), & \text{if } p_{k,1}[n] - \lambda_{k,n}r_{u,k}[n] + \eta_{k,n} = 0 \\ 0, & \text{if } p_{k,1}[n] - \lambda_{k,n}r_{u,k}[n] + \eta_{k,n} > 0 \end{cases}$$

**Proof:** See Appendix A.

The solutions to the subproblems (L2)-(L6) are given in Lemmas 4-8, respectively, and the proofs of these problems are omitted here due to be similar KKT method applied in the Lemma 3.

**Lemma 4:** By solving subproblem (L2) with KKT, the optimal solution can be denoted as

$$E_{k,2}[n] = p_{k,2}[n]t_{k,2}[n],$$

$$p_{k,2}[n] = \left[ \frac{\mu_{k,n}B_0}{\ln 2} - \frac{1}{\gamma_0} \right]^{\frac{1}{\gamma_0}}_{\nu_{k,n} - \omega_{k,n}}$$

$$t_{k,2}[n] = \begin{cases} \delta_t, & \text{if } p_{k,2}[n] - \lambda_{k,n}r_{u,k}[n] + \eta_{k,n} < 0 \\ \epsilon(0, \delta_t), & \text{if } p_{k,2}[n] - \lambda_{k,n}r_{u,k}[n] + \eta_{k,n} = 0 \\ 0, & \text{if } p_{k,2}[n] - \lambda_{k,n}r_{u,k}[n] + \eta_{k,n} > 0 \end{cases}$$

**Lemma 5:** By solving subproblem (L3) with KKT, the optimal solution can be denoted as

$$E_{k,3}[n] = p_{k,3}[n]t_{k,3}[n],$$

$$p_{k,3}[n] = \left[ \frac{\nu_{k,n}B_0}{\ln 2} - \frac{1}{\gamma_0} \right]^{\frac{1}{\gamma_0}}_{\nu_{k,n} + \omega_{k,n}}$$

$$t_{k,3}[n] = \begin{cases} \delta_t, & \text{if } p_{k,3}[n] - \lambda_{k,n}r_{u,k}[n] + \eta_{k,n} < 0 \\ \epsilon(0, \delta_t), & \text{if } p_{k,3}[n] - \lambda_{k,n}r_{u,k}[n] + \eta_{k,n} = 0 \\ 0, & \text{if } p_{k,3}[n] - \lambda_{k,n}r_{u,k}[n] + \eta_{k,n} > 0 \end{cases}$$

**Lemma 6:** By solving subproblem (L4) with KKT, the optimal solution can be denoted as

$$l_{a,k}[n] = \delta_t \left[ \frac{\omega_{k,n} - \lambda_{k,n}}{3\kappa c_h} \right]^{\frac{1}{\gamma_h}}_{\mu_{k,n}}$$

**Lemma 7:** By solving subproblem (L5) with KKT, the optimal solution can be denoted as

$$l_{a,k}[n] = \begin{cases} \delta_t \left[ \frac{\omega_{k,n} - \lambda_{k,n}}{3\kappa c_h} \right]^{\frac{1}{\gamma_h}}_{\mu_{k,n}}, & \text{if } \omega_{k,n} - \lambda_{k,n} \geq 0 \\ 0, & \text{if } \omega_{k,n} - \lambda_{k,n} < 0 \end{cases}$$

**Lemma 8:** By solving subproblem (L6) with KKT, the optimal solution can be denoted as

$$l_{a,k}[n] = \begin{cases} 0, & \text{if } \mu_{k,n} + \nu_{k,n} - \omega_{k,n} > 0 \\ a, & \text{if } \mu_{k,n} + \nu_{k,n} - \omega_{k,n} = 0 \end{cases}$$

where $a$ represent any non-negative constant.

Based on the duality method, it can be seen from Lemma 3-5 that the offloading strategy depends on the channel quality between the UAV and TDs or that between the UAV and AP. For example, the expression (35b) indicates that the UAV would...
help TDs forward the task bits to AP if the distance between the UAV and AP is smaller than a threshold, i.e., \( d_{th}[n] \leq \sqrt{\frac{\nu_{th} B_0}{h_2} \gamma_1} \). Moreover, from Lemma 6 and 7, we can know that TDs would choose to perform bits offloading to UAV for computing when the local computation task exceed the amount of \( \sqrt{\frac{\lambda_{th} \delta_{th}}{3 N c_0}} \). Otherwise, the TDs only operate local computing.

### B. Obtaining \((\lambda, \mu, \nu, \omega, \eta)\) by Solving Problem (D2)

After obtaining \((L^*, t^*, E^*)\) for given \((\lambda, \mu, \nu, \omega, \eta)\), we can then obtain the optimal dual variables by solving problem (D2), denoted by \((\lambda^*, \mu^*, \nu^*, \omega^*, \eta^*)\). Considering problem (D2) is non-differentiable in general, this motivates us to use the ellipsoid method [33] to solve problem (D2). Specifically, the subgradient of the objective function can be represented by \((\Delta \lambda^T, \Delta \mu^T, \Delta \nu^T, \Delta \omega^T, \Delta \eta^T)\), in which the vectors \( \Delta \lambda, \Delta \mu, \Delta \nu, \Delta \omega, \Delta \eta \) are respective given as

\[
\Delta \lambda = \sum_{i=1}^{n} l_{a,k}[i] - \sum_{i=1}^{n} t_{k,1}[i] r_{a,h}(E_{k,1}[i] / t_{k,1}[i]), \forall k, n, \tag{39a}
\]

\[
\Delta \mu = l_{a,k}[n] - t_{k,2}[n] r_{a,h}(E_{k,2}[n] / t_{k,2}[n]), \forall k, n, \tag{39b}
\]

\[
\Delta \nu = l_{a,k}[n] - t_{k,3}[n] r_{a,h}(E_{k,3}[n] / t_{k,3}[n]), \forall k, n, \tag{39c}
\]

\[
\Delta \omega = L^{min}_{k,n} - l_{a,k}[n] - l_{k,1}[n] - l_{k,2}[n], \forall k, n, \tag{39d}
\]

\[
\Delta \eta = t_{k,1}[n] + t_{k,2}[n] + t_{k,3}[n] - \delta_i, \forall k, n, \tag{39e}
\]

### C. Constructing Optimal Solution to Problem (P2)

Due to the nonuniqueness of \( t_{k,m}[n], m = \{1, 2, 3\} \) and \( l_{a,k}[n] \), an extra step is needed to construct the optimal solution to problem (P2). From Lemma 3-8, the obtained solutions \( p_{k,m}^*[n], m = \{1, 2, 3\} \), \( t_{k,m}[n] \), \( l_{a,k}[n] \) are unique. By substituting these parameters in problem (P2), we have

\[
\min_{t_{a,k}[n], L_{k,n}} \sum_{n=0}^{N} \sum_{k=1}^{K} E_{k,1}[n] + E_{k,2}[n] + E_{k,3}[n] \tag{40a}
\]

s.t. \((23g)-(23l))

\[
\sum_{n=0}^{N} \sum_{k=1}^{K} l_{a,k}[n] \leq \sum_{n=0}^{N} t_{k,1}[n] r_{a,h}(p_{k,1}[i]), \forall k, n, \tag{40c}
\]

\[
l_{a,k}[n] \leq t_{k,2}[n] r_{a,h}(p_{k,2}[i]), \forall k, n, \tag{40d}
\]

\[
l_{a,k}[n] \leq t_{k,3}[n] r_{a,h}(p_{k,3}[i]), \forall k, n, \tag{40e}
\]

\[
l_{a,k}[n] + l_{a,k}[n] + l_{k,1}[n] \geq L_{k,n}^{min}, \forall k, n. \tag{40f}
\]

By solving the linear programming problem (40), the optimal solution to primal problem (P2) is obtained. The details for solving problem (P2) is summarized in Algorithm 1.

---

### Algorithm 1 A dual algorithm to optimally solve (P2)

1. Initialization: \( \lambda, \mu, \nu, \omega, \eta \), and the ellipsoid.
2. repeat
3. Based on Lemma 3-8, obtain \( L^*, t^*, E^* \).
4. By solving problem (D2), obtain the subgradients of the objective functions and constraints.
5. Update \( \lambda, \mu, \nu, \omega, \eta \) based on ellipsoid method.
6. until \( \lambda, \mu, \nu, \omega, \eta \) converge.
7. Let \((\lambda^*, \mu^*, \nu^*, \omega^*, \eta^*)\) = \((\lambda, \mu, \nu, \omega, \eta)\).
8. Obtain \( p_{k,m}^*[n], m = \{1, 2, 3\}, t_{k,m}[n], l_{a,k}[n] \) based on Lemma 3-8, and then obtain optimal \( t_{k,m}[n], m = \{1, 2, 3\} \) and \( l_{a,k}[n] \) by solving problem (40).
symbol and νk, as expressed in (46) and (47), respectively.

Similarly, for any given local point \{q_j[n]\}, the RHHs of the inequalities in (41b) and (41c) can be also lower-bounded. The corresponding lower bound functions can be derived, as expressed in (46) and (47), respectively.

By replacing the derived lower bound functions and the approximately convex expression into problem (P3), we can obtain

\[
\min_{L, \mathbf{Q}, v_n, u_n} \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{k_u (c_u l_u k [n])^3}{\delta^3_l} + \frac{k_h (c_h l_h k [n])^3}{\delta^3_l} + w_\delta \sum_{n=1}^{N} \mathbf{P}_{appr}(v_n)
\]

s.t. (18a)-(18c), (20), (23e)-(23g), (23j),

\[
\sum_{i=1}^{n} l_h k [i] \leq \sum_{i=1}^{n} l_h k [i] B_0 \varphi_k b_k [i], \forall k, n \tag{49a}
\]

\[
l_a k [n] \leq l_a k b [n] B_0 \varphi_k b_k [n], \forall k, n \tag{49b}
\]

\[
l_a k [n] \leq l_a k b [n] B_0 \varphi_k b_k [n], \forall k, n \tag{49c}
\]

\[
v_n^2 \geq \|n[n]\|^2, n \in N \tag{49d}
\]

\[
\lambda_k \geq \frac{1}{\omega_n}, n \in N \tag{49e}
\]

It can be readily proved that the optimal solution always makes equality hold in (49b), (49c) and (49e). Also, the equality must holds in the causality condition (49a) for n = N. Hence, the problem (P3.1) is equivalent to (P3). Obviously, Problem (P3.1) is convex that can be solved by standard convex optimization tools, such as CVX [34].

In summary, an overall iterative algorithm that jointly optimizes computation bits allocation, power allocation, time slot scheduling and UAV trajectory can be derived to solve the primal problem (P1), as summarized in Algorithm 2. Algorithm 2 consists of the duality method and SCA technology, at least a locally optimal solution always can be achieved by the proposed joint optimization algorithm.

Algorithm 2: The overall iterative algorithm to solve (P1)

1. Given UAV initial local point \{q_j[n]\}, \{v_{n,j}\} and \{u_{n,j}\}, let iteration j = 0.
2. repeat
3. With \{q_j[n]\}, solve (P2) based on Algorithm 1 and obtain \{p^*, E^*, t^*\}.
4. With \{p^*, E^*, t^*\} and \{q_j[n]\}, solve problem (P3.1), and obtain optimized trajectory denoted by \{q_j^*[n]\}, \{v_{n,j}^*\} and \{u_{n,j}^*\} via CVX.
5. Update \{q_j[n+1]\} \leftarrow \{q_j^*[n]\}, \{v_{n,j}\} \leftarrow \{v_{n,j}^*\}, \{u_{n,j}\} \leftarrow \{u_{n,j}^*\}.
6. Update j \leftarrow j + 1.
7. until The objective value converges.

Here, we briefly give the complexity analysis for the proposed algorithms. For each iteration of Algorithm 2, it consists of solving Algorithm 1 and optimizing UAV trajectory with CVX. The computation complexity of Algorithm 1 mainly depends on the loop, i.e., step 3) to step 5) of Algorithm 1. Note that the complexity of ellipsoid method is \mathcal{O}(K^2N^2) [32] [33]. Thus the complexities of step 3), 4) and 5) of Algorithm 1 are \mathcal{O}(K^N), \mathcal{O}(K^N) and \mathcal{O}(K^2N^2), respectively. As a result, the total complexity for the Algorithm 1 is \mathcal{O}(K^4N^4).

V. Numerical Results

In this section, the numerical results are presented to validate our proposed design. The vector \mathbf{L}_m \in \mathbb{R}^{1 \times 3} is utilized to represent the set of required computation bits, in which the k\textsuperscript{th} entry stands for the required computation task for TD k in per time slot. The details of parameter setup are shown in Table I.

<table>
<thead>
<tr>
<th>Symbolic Meaning</th>
<th>Symbolic and Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude of UAV</td>
<td>H = 20 m</td>
</tr>
<tr>
<td>Amount of TDs</td>
<td>K = 3</td>
</tr>
<tr>
<td>Maximum speed</td>
<td>V_{max} = 20 m/s</td>
</tr>
<tr>
<td>Initial location of UAV</td>
<td>q_0 = [−20, −20] m</td>
</tr>
<tr>
<td>Final location of UAV</td>
<td>q_F = [20, −20] m</td>
</tr>
<tr>
<td>Time slot size</td>
<td>\delta_t = 0.2 s</td>
</tr>
<tr>
<td>Maximum instantaneous power of each users for offloading</td>
<td>P_{max}^u = 35 dB</td>
</tr>
<tr>
<td>Maximum instantaneous power of UAV</td>
<td>P_{max} = 35 dB</td>
</tr>
<tr>
<td>Noise power spectrum density</td>
<td>\delta_b = 50 dB</td>
</tr>
<tr>
<td>Reference channel power</td>
<td>B = 10 MHz</td>
</tr>
<tr>
<td>Maximum CPU frequency of each TD</td>
<td>f_{CPU} = 2 GHz</td>
</tr>
<tr>
<td>Maximum CPU frequency of UAV</td>
<td>f_{CPU} = 3 GHz</td>
</tr>
<tr>
<td>Required CPU cycles per bit computation at TD</td>
<td>c_u = 10^3 cycles/bit</td>
</tr>
<tr>
<td>Required CPU cycles per bit computation at UAV</td>
<td>c_u = 10^3 cycles/bit</td>
</tr>
<tr>
<td>CPU capacitance coefficient of each TD</td>
<td>k_u = 10^{-27}</td>
</tr>
<tr>
<td>CPU capacitance coefficient of UAV</td>
<td>k_u = 10^{-27}</td>
</tr>
<tr>
<td>Weight</td>
<td>\omega = 0.01</td>
</tr>
<tr>
<td>Tip speed of the rotor blade</td>
<td>U_{tip} = 120 m/s</td>
</tr>
<tr>
<td>Rotor disc area</td>
<td>A = 0.503 m^2</td>
</tr>
<tr>
<td>Air density</td>
<td>\rho = 1.225 (kg/m^3)</td>
</tr>
<tr>
<td>Rotor solidity</td>
<td>s = 0.05</td>
</tr>
<tr>
<td>Fuselage drag ratio</td>
<td>\delta_0 = 0.3</td>
</tr>
<tr>
<td>Mean rotor induced velocity in hover</td>
<td>v_0 = 4.03</td>
</tr>
<tr>
<td>Blade profile power in hovering status</td>
<td>P_l = 158.76 w</td>
</tr>
<tr>
<td>Induced power in hovering status</td>
<td>P_l = 88.63 w</td>
</tr>
</tbody>
</table>

Fig. 3. The convergence of the proposed algorithm for period T = 6 s.

Note that in order to illustrate the effectiveness of our proposed design, several other benchmark cases are designed as follows.

1) Straight Flight design. In this case, the UAV flies from the given initial location to final location following a straight trajectory.
2) No AP design. In this case, the task bits of TDs are computed without AP cooperation.
3) Only Relaying design. In this case, the UAV can only act as a relay to assist task bits transmit from TDs to AP.
4) No UAV Cooperation design. In this case, the task bits can only be computed locally at each TD. Note that for
\[ \varphi_{k,1}^{b}[t] = \log_2 \left( 1 + \frac{p_{k,1}^{b}[t] \gamma_0}{||q_j[t] - w_k||^2 + H^2} \right) - \frac{\log_2(e) p_{k,1}^{b}[t] \gamma_0 (||q_j[t] - w_k||^2 - ||q_j[t] - w_k||^2)}{(||q_j[t] - w_k||^2 + H^2)(||q_j[t] - w_k||^2 + H^2 + p_{k,1}^{b}[t] \gamma_0)}. \]

\[ \varphi_{k,2}^{b}[n] = \log_2 \left( 1 + \frac{p_{k,2}^{b}[n] \gamma_0}{||q_j[n] - w_k||^2 + H^2} \right) - \frac{\log_2(e) p_{k,2}^{b}[n] \gamma_0 (||q_j[n] - w_k||^2 - ||q_j[n] - w_k||^2)}{(||q_j[n] - w_k||^2 + H^2)(||q_j[n] - w_k||^2 + H^2 + p_{k,2}^{b}[n] \gamma_0)}. \]

\[ \varphi_{k,3}^{b}[n] = \log_2 \left( 1 + \frac{p_{k,3}^{b}[n] \gamma_0}{||q_j[n] - w_n||^2 + H^2} \right) - \frac{\log_2(e) p_{k,3}^{b}[n] \gamma_0 (||q_j[n] - w_n||^2 - ||q_j[n] - w_n||^2)}{(||q_j[n] - w_n||^2 + H^2)(||q_j[n] - w_n||^2 + H^2 + p_{k,3}^{b}[n] \gamma_0)}. \]
respectively. Then, an iterative algorithm was proposed to solve the primal problem. The numerical results validated the effectiveness of our proposed algorithm and showed the superiority of our proposed design, as compared to the other benchmark designs.

APPENDIX A
PROOF OF LEMMA 3
The Lagrangian of subproblem (L1) is given as
\[
\mathcal{L}_1(\Xi) = E_{k,1}[n] - \lambda_{k,n}t_{k,1}[n]r_{ub} \left( \frac{E_{k,1}[n]}{t_{k,1}[n]} \right) + \eta_{k,n}t_{k,1}[n] - aE_{k,1}[n] + b(E_{k,1}[n] - t_{k,1}[n]P_{umax}^k) - c_{k,1}[n] + d(t_{k,1}[n] - \delta_t)
\]
(50)
where \( \Xi \) is the set denoted by \( \Xi = (a_k^1, b_k^1, c_k^1, d_k^1) \), with \( a_k^1, b_k^1, c_k^1 \) and \( d_k^1 \) representing the non-negative Lagrange multipliers with regard to the \( E_{k,1}[n] \geq 0, E_{k,1}[n] \leq t_{k,1}[n]P_{umax}^k, t_{k,1}[n] \geq 0 \) and \( t_{k,1}[n] \leq \delta_t \), respectively. Thus, the derivations of \( \mathcal{L}_1(\Xi) \) with respect to \( E_{k,1}[n] \) can be expressed as
\[
\frac{\partial \mathcal{L}_1(\Xi)}{\partial E_{k,1}[n]} = 1 - \lambda_{k,n} \frac{B_0}{\ln 2} \frac{\gamma_0}{1 + \frac{E_{k,1}[n] \gamma_0}{t_{k,1}[n]}} - a_k^1 + b_k^1
\]
(51)
Based on KKT, the complementary slackness conditions are given by \( a_k^1 E_{k,1}[n] = 0, b_k^1 E_{k,1}[n] - t_{k,1}[n] P_{umax}^k = 0, c_{k,1} t_{k,1}[n] = 0 \) and \( d_k^1 (t_{k,1}[n] - \delta_t) = 0 \). Let the derivation \( \frac{\partial \mathcal{L}_1(\Xi)}{\partial E_{k,1}[n]} = 0 \), we can obtain the equation (33a) and (33b). By substituting (33a) into subproblem (L1) the optimal \( t_{k,1}[n] \) can be easily obtained. Hence, the Lemma is proved.

REFERENCES


