A model for the strength analysis of high-strength concrete (HSC) columns subjected to eccentric loading is proposed. The model is based on a stability analysis of pin-ended columns using the theoretical sinusoidal equation for the deflected shape of the column. The reduction in column stiffness as the axial load increases, representing the basic characteristic of the inelastic response of columns, is considered subject to equilibrium conditions, compatibility requirements, and constitutive relationships for the concrete and reinforcement. The tension-stiffening effect was taken into consideration. The column integrity is limited by either the material or the instability mode of failure. The method was applied to a wide range of experimental data and was compared with the Egyptian, European, and American building codes of practice. The ultimate strength predicted by the proposed model showed excellent agreement with the test results and was in good agreement with the codes of practice. The mean predicted-to-experimental ultimate load ratio was 0·94, with a coefficient of variation of 10·8%.

**Notation**

- $A_{xx}$, $A_{yy}$: total cross-sectional areas of transverse bars perpendicular to x- and y-axes, respectively
- $C_{in}$: end effect factor
- $c$: factor depending on the curvature distribution
- $c_x$, $c_y$: dimensions of the concrete core parallel to the x- and y-axes, respectively
- $E_c$: elastic modulus of concrete
- $E_s$: secant modulus of concrete
- $EI$: flexural rigidity of column section
- $e$: initial eccentricity of applied load, $P$
- $f'_c$: ultimate compressive strength of plain concrete obtained from standard cylinder test
- $f_c$: stress in concrete
- $f_{cc}$: confined concrete compressive strength based on $f'_c$
- $f_{co}$: stress in transverse reinforcement at maximum strength of confined concrete
- $f_{hay}$: yield strength of ties
- $f_{he}$: effective confinement pressure applied on concrete core
- $f_t$: tensile strength of concrete
- $h$: side length of column in buckling direction
- $I_x$: confinement index ($= f_{he}/f'_t$)
- $I_{cc0}$: effective confinement index
- $i$: radius of gyration of column cross-section
- $k$: initial slope and curvature of ascending branch
- $k_c$: confinement effectiveness coefficient, depending on the proportions of the column cross-section and the amount and configuration of reinforcement
- $l$: effective column length
- $M_{Ed}$: total design bending moment of non-sway slender reinforced concrete columns
- $M_{Ed}$: first-order moment
- $n = (P/EI)^{1/2}$: ultimate load on column ($n2EI/l^2$)
- $P_e$: Euler elastic critical buckling load
- $P_{p/e}$: predicted-to-experimental ultimate load ratio
- $[1/r]$: curvature calculated based on empirical equations taking into consideration the level of axial load and creep effects
- $s$: centre-to-centre spacing between stirrups
- $s_c$: clear spacing between stirrups
- $x$: deflection of column
- $\beta$: factor depending on the distribution of first- and second-order moments
- $\Delta$: deflection, independent of the load characteristics
- $\delta$: moment magnifier factor
- $e_{cc}$, $e_{co}$: axial strains in confined and unconfined concrete, respectively
- $e_{50c}$, $e_{50u}$: post-peak axial strain in confined concrete, axial strain in unconfined concrete
- $\nu_{cc}$: secant Poisson coefficient at maximum peak stress
- $\rho_c$: longitudinal reinforcement ratio in core section
- $\rho_{lc}$: longitudinal reinforcement ratio
- $\sum_{i=1}^{n}w_i$: sum of the squares of the clear spacing between longitudinal bars
**Introduction**

The use of high-strength concrete (HSC) in reinforced concrete columns reduces the column proportions, heightening the adverse effects of the slenderness and potential instability on the column capacity. The significance of slenderness effects in HSC columns has caused concern regarding the applicability of current building code requirements for the design of HSC slender columns. The methods available in the literature for the strength analysis of slender columns are generally based on a simplified non-linear analysis that approximates the column deflection to a sine wave function with either constant (Chuang and Kong, 1998; Kuzmanovic, 2014; Lloyd and Rangan, 1996) or variable (Bažant and Xiang, 1997; Mendis, 2000) wavelength (effective length of the column). The beneficial influence of confinement of the concrete core on the column response (Galeota et al., 1992) (i.e. deformation and strength) has generally been ignored. Commonly, the available methods adopt stress–strain models for unconfined concrete. Probabilistic analyses of the modelling errors of various methods for strength analysis of slender columns (Zhou and Hong, 2001) have indicated that modelling errors are sensitive to the adopted concrete stress–strain relationship.

Based on extensive investigation of the response of HSC columns, Légeron and Paultre (2003) developed a stress–strain model for confined HSC that considers the effects of concrete strength and transverse and longitudinal reinforcement parameters on the significance of confinement. The strength of confined concrete is determined based on an effective confinement pressure that depends on the stress of transverse reinforcement at the peak strength of concrete and on the configuration of the restrained concrete core. The results of non-linear finite-element analysis of slender columns under eccentric loads (Claeson and Johansson, 1999; Kim and Yang, 1995; Kottb et al., 2015) evince the efficiency of the model in assessing the significance of confinement. Experimental and theoretical models for strength analysis of normal-strength and transverse and longitudinal reinforcement. The stress-strain relationship of the concrete.

The proposed model was applied to relevant test data from the literature to determine its level of accuracy in dealing with any combination of column parameters. The comparison of the results involved the column capacity and the load-deflection characteristics. In addition, the applicability of current building codes ECP-203 (HRBC, 2007), Eurocode 2 (EC2) (BSI, 2004) and ACI 318 (ACI, 2014) to the strength analysis of HSC columns was examined with respect to the proposed model.

**Code provisions**

The design of slender columns is based on the straining actions resulting from a second-order analysis of the structure, taking into account the effects of material and geometrical non-linearities including creep effects. For the sake of simplicity, the codes specify two alternative methods to account for the second-order bending moment.

The first method, based on nominal stiffness, is EC2-1 or the so-called moment magnifier method. EC2 (BSI, 2004) specifies the total design bending moment of non-sway slender reinforced concrete columns ($M_{ED}$) to be:

1. \[ M_{ED} = M_{OD} \left[ 1 + \frac{\beta}{(P_c/P) - 1} \right] \]

ACI 318 (ACI, 2014) employs a moment magnifier factor, $\delta$, to account for second-order effects assuming implicitly a half-wave sinusoidal shape for the deflection curve of the column:

2. \[ \delta = \frac{C_m}{1 - (P/P_c)} \geq 1.0 \]

Both EC2 and ACI 318 specify empirical equations for determination of the $EI$ value, allowing for the effects of cracking, creep, and non-linearity of the stress–strain relationship of the concrete.

The second method suggested in EC2, called EC2-2, is based on the nominal curvature, where an additional
second-order bending moment is calculated assuming the deflection $\Delta$ to be

$$\Delta = \frac{P}{c} \left( \frac{l}{r} \right)$$

The solution for the ultimate load in these equations requires an iterative procedure. On the other hand, the Egyptian code ECP-203 (HRBC, 2007) considers the second-order effects through an additional load eccentricity, $\Delta$, specified to be independent of the load characteristics. The additional eccentricity for rectangular columns is defined as

$$\Delta = h \left( \frac{0.3l}{i} \right) / 2000; \quad l/i \geq 50$$

Overall, it can be seen that no refinement of the codes’ design methods is feasible without implementing information on the characteristics of concrete as a material.

**Proposed method of analysis**

A method for the strength analysis of eccentrically loaded pin-ended braced columns was developed based on stability criteria. The method considers equilibrium and compatibility requirements and the material properties at the critical section of the column. The method is applicable to short to extremely slender columns, accepting that the column integrity is limited by material and/or instability modes of failure.

**Column deflection model**

Even though the column response is inelastic, a good approximation may be obtained by utilising the elastic deflection equation of the column, although with an appropriate flexural rigidity. According to Kim and Yang (1995), a suitable model for flexural rigidity is essential to assess the actual column response. Referring to Figure 1, the second-order differential equation for a column with pinned ends and subjected to eccentric load is

$$\frac{d^2y}{dx^2} = -n^2(y + e)$$

The solution of this equation is given in texts on the mechanics of materials (Hearn, 1997) as

$$y + e = e \tan(nl/2) \sin nx + e \cos nx$$

It is clear that the deflection curve is sinusoidal. Thus, the deflection is maximum at the column mid-height, $x = l/2$, and is given as

$$\Delta = e \left[ \sec(nl/2) - 1 \right]$$

The term $nl/2$ may be expressed in terms of the critical buckling load as follows.

$$n = \sqrt{\frac{P}{2EI}}$$

$$P_c = \pi^2 EI / l^2$$

Then

$$nl/2 = (\pi/2) \sqrt{P/P_c}$$

The term $nl/2$ indicates that the column deflection is dependent on the eccentricity, the level of applied load and the buckling load. Incidentally, the equation given in ACI 318 (ACI, 2014) for the moment magnifier factor (Equation 2) is an approximate version of Equation 7 (Park and Paulay, 1975). In non-linear analysis, the flexural rigidity is variable, depending on the cracking intensity that, in turn, depends on the load level (cracked section modelling). Therefore the exact equation includes, via the buckling load, the flexural rigidity of the column section as a variable that is dependent on the load level. It is worth mentioning that the flexural rigidity varies along the column length. The buckling load was assumed to be subject to the tangent flexural rigidity of the effective section at the column mid-height. It is formulated by dividing the critical section into steel and concrete layers through its depth. This layering approach permits a variation in strain and...
stress resultants across the section depth. Strain compatibility across the section depth is based on the assumption that plane sections remain plane after bending. The membrane actions of the layers are integrated through the depth to obtain the flexural rigidity.

**Material model**

**Modelling of concrete**

Reported data demonstrate the necessity of considering two models for concrete under compression in a column cross-section – one for the unconfined cover and the other for the confined core. Otherwise, an overly conservative post-peak behaviour is expected (Claeson and Johansson, 1999). The material model adopted for concrete in compression is based on the model suggested by Légeron and Paultre (2003), which is an update of the model presented by Cusson and Paultre (1995). Figure 2 shows the stress–strain relationships for unconfined and confined concrete, respectively representing the concrete cover and the core of the column section. The ascending branch of the stress–strain relationship of confined concrete is based on a relationship originally proposed by Popovics (1973).

11. \[ f_c = f_{co} \left[ \frac{k(k_c/k_{co})}{k - 1 + (k_c/k_{co})^2} \right] \]

in which

12. \[ f_{co} = f_{co} \left[ 1 + 2.4(I_0)^{0.7} \right] \]

13. \[ \varepsilon_{co} = \varepsilon_{co} \left[ 1 + 35(I_0)^{1.2} \right] \]

14. \[ k = \frac{E_c}{E_c - (f_{co}/\varepsilon_{co})} \]

The descending branch is based on the model proposed by Fafitis and Shah (1985)

15. \[ f_c = f_{ce} \exp \left[ k_1(k_c/k_{ce})^{k_2} \right] \]

in which

16. \[ k_1 = \ln(0.5)/(\varepsilon_{ce}/\varepsilon_{co})^{k_2} \]

17. \[ k_2 = 1 + 25/(I_{co})^2 \]

18. \[ \varepsilon_{ce} = \varepsilon_{50c} + (1 + 60I_{co}) \]

where the coefficients \( k_1 \) and \( k_2 \) control the general slope and the curvature of the descending branch, respectively. Referring to Figure 2, \( \varepsilon_{ce} \) and \( \varepsilon_{50c} \) are the axial strains in confined and unconfined concrete at which the stress drops to 0.5\( f_{co} \) and 0.5\( f_{ce} \), respectively. Herein, \( \varepsilon_{50c} \) was taken as 0.004. The effective confinement index, \( I_{co} \), is evaluated at the post-peak strain (\( \varepsilon_{ce} \)) assuming the reinforcement bars are yielded.

The aforementioned constitutive model is shown to depend in essence on the effective confinement pressure, \( f_{co} \). Accepting that the confinement pressure developed by transverse reinforcement is non-uniform, Légeron and Paultre (2003) assumed the effective confinement pressure to be

19. \[ f_{co} = \frac{k_1 k_2 f_h}{2} \left( \frac{A_{ux} + A_{uy}}{c_x + c_y} \right) \]

where

20. \[ k_{ie} = \frac{1 - \left( \sum w_j \right)^2 / 6c_x c_y}{\left[ 1 - (x_c/2c_x) \right] \left[ 1 - (x_c/2c_y) \right]} \]

There are two alternatives to compute the stress in the transverse reinforcement, \( f_{hu} \), at peak strength of confined concrete. The first alternative is the iterative approach suggested by Cusson and Paultre (1995) in which the corresponding strain, \( \varepsilon_{ep} \), needs to be estimated as

21. \[ \varepsilon_{ep} = \varepsilon_{co} \left( 1 - \frac{f_{ep}}{f_{co}/\varepsilon_{co}} \right) \]

considering the secant Poisson’s coefficient at maximum peak stresses (\( \nu_{ep} \)) to be equal to 0.43. On the other hand, the direct
approach suggested by Légeron and Paultre (2003), based on a wide range of tested columns, assumes that $f_h$ is equal to the yield strength of the ties, $f_{hy}$, if $\eta$ reaches a value of 10 where

$$22. \quad \eta = \frac{f'_c}{k_z E_{co}} \left( \frac{c_x + c_y}{A_{dx} + A_{dy}} \right)$$

otherwise

$$23. \quad f_h = \frac{0.25 f'_c}{(k_z s)} \left[ \frac{(c_x + c_y)}{(A_{dx} + A_{dy})} \right] (\eta - 10) \geq 0.43 E_{co}, \quad E_s$$

Modelling of reinforcement

The stress–strain relationship for the longitudinal reinforcement was modelled by an elastic–perfectly plastic approximation identical in tension and compression. Perfect bond between the reinforcing bars and concrete was assumed.

Analysis procedure

The deflection response of the column, including the post-peak response, is determined by solving for the deflection condition(s) as the load is incremented. For a given load level, the solution for the deflection involves an iterative procedure to comply with the equilibrium and compatibility requirements along with the constitutive relationships for concrete and reinforcement at the critical section of the column. The procedure begins by estimating the strain profile at the critical section and proceeds to consider the resultant stress condition to solve for the column deflection using Equation 7. Then, the conditions of equilibrium of the internal and external forces are examined and the estimated strain profile is adjusted accordingly until convergence is achieved. The convergence criterion was based on unbalanced forces with a convergence tolerance of 0.005. The

Table 1. Column specimens tested by Lloyd and Rangan (1996) – details and ultimate response

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$f'_c$: MPa</th>
<th>$\rho_l$: %</th>
<th>$l_i$</th>
<th>$e/h$</th>
<th>$P_0$: kN</th>
<th>$\Delta_l$: mm</th>
<th>$P_0'$: kN</th>
<th>$\Delta_l'$: mm</th>
<th>$P_{EC1}$/$P_t$</th>
<th>$P_{EC2}$/$P_t$</th>
<th>$P_{EC3}$/$P_t$</th>
<th>$P_{AC1}$/$P_t$</th>
<th>$\text{CoV}$: %</th>
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<tr>
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<td>1250</td>
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<td>583</td>
<td>15.2</td>
<td>0.81</td>
<td>1.09</td>
<td>0.78</td>
<td>0.93</td>
<td>0.89</td>
</tr>
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<td>12.9</td>
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<tr>
<td>VIIIC</td>
<td>92</td>
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$*The$ $yield$ $strength$ $of$ $longitudinal$ $reinforcement$ $was$ $430$ $MPa$

$*The$ $cube$ $strength$ $of$ $concrete$ $was$ $taken$ $as$ $1.25f'_c$
load level beyond which convergence was not fulfilled was considered the ultimate load of the column. Note that the analysis of slender columns using the methods specified by EC2 and ACI 318 entails a trial-and-error procedure to determine the ultimate load.

**Experimental verification**

The proposed method was applied to extensive test data from the literature to examine its credibility in predicting the response of HSC columns. Of particular interest were the load-carrying capacity of the column and the load-deflection characteristics. The ultimate load predictions were also compared with the results following the ECP-203, EC2 and ACI 318 code provisions for columns under uniaxial eccentric compression. The strength reduction factors specified by the codes were adopted as unity.

Specimens investigated by Lloyd and Rangan (1996)
The behaviour of HSC columns under eccentric compression was extensively investigated by Lloyd and Rangan (1996). The column parameters were the concrete compressive strength, longitudinal reinforcement ratio, load eccentricity and column cross-section. Five series were chosen for analysis. The volumetric ratio of transverse reinforcement was 0·46% and its yield strength was 450 MPa. The analytical results are compared with the experimental results in Table 1.

Referring to Table 1, excellent agreement was achieved between the predicted and observed ultimate responses. The analysis reflected the essence of the test results that an increase in the initial eccentricity ratio (\(e/h\), where \(e\) is the eccentricity and \(h\) is the length of column's cross-section) resulted in a decrease in the ultimate load and an increase in the column deflection at failure. The mean predicted-to-experimental ultimate load ratio \(P_t/P_u\) was 0·93, with a coefficient of variation (CoV) of 9·80%. The predictions of the column's deflection compare well with the experimental results, not only at the ultimate load level but also throughout the loading range, as revealed by the load-deflection responses shown in Figure 3. There is a tendency, however, to overestimate the post-cracking deflections for specimens with \(e/h \geq 0\·30\). The tension-stiffening effect may be underestimated in the present analysis, resulting in a decrease in the column stiffness and consequently an increase in the post-cracking deformations; however, it becomes more influential as \(e/h\) increases. For specimens with \(e/h = 0\·10\), the analytical load-deflection response was found to be satisfactory.

The ultimate load predicted by EC2-2 (the method based on nominal curvature) was closer to the experimental value than the ECP-203 results. The mean \(P_{EC2}/P_u\) ratio was 0·94, with a CoV of 7·0% whereas the mean \(P_{ECP}/P_u\) ratio was 1·18 with a CoV of 9·80%. It should be noted that the stress-strain relationship provided by EC2 accounts for HSC of compressive strength up to 100 MPa. Besides, ECP-203 neglects the additional eccentricity for specimens with \(li < 50\). EC2-1 (the method based on nominal stiffness) tended to underestimate the ultimate loads for columns with \(e/h \geq 0\·3\), while the ACI 318 procedure slightly overestimated the ultimate loads for columns with \(e/h \leq 0\·1\). This may be attributed to the methods proposed for assessing the column flexure rigidity. The mean \(P_{EC2,1}/P_u\) ratio was 0·85 with a CoV of 14·9% whereas the mean \(P_{ACI}/P_u\) ratio was 0·97 with a CoV of 9·70%. In practice, such a discrepancy in the results of the building codes is expected to yield a non-uniform safety margin in column design.

Kim and Yang (1995) reported test results on 30 tied columns of 80 mm square cross-section and slenderness ratios of 10, 60 and 100. The initial eccentricity of the applied load was 24 mm. Three strengths of concrete (25-5, 63-5 and 86-2 MPa) and two ratios of longitudinal steel (2.0% and 4.0%) were used. The volumetric ratio of transverse reinforcement was 0.044% and its yield strength was 250 MPa. As shown in Table 2, only HSC specimens were considered in the analysis.

Table 2 shows that the correlation with ultimate load predictions was satisfactory, with a mean value of $P_t/P_i$ of 0.93 and a CoV of 13.6%. The rather high variation in the test deflections at failure was adequately predicted, particularly for columns with $l/i = 100$ and a longitudinal reinforcement ratio of 4%. The mean $P_{ECP}/P_t$ ratio was 0.81 with a CoV of 10.6%. Except for ECP-203, the iterative process in all the procedures adjusts the neutral axis position as it decreases by increasing the longitudinal reinforcement content, resulting in higher mid-height deflections. On the other hand, the methods based on nominal stiffness (EC2-1 and ACI 318) significantly underestimated the predicted ultimate loads for columns with $l/i = 100$ and a longitudinal reinforcement ratio of 4%. However, excellent results were obtained using the proposed method. The tension-stiffening effect appears to be significant in affecting eccentric columns, particularly when the bending moment is dominant. The provisions set by EC2-2 and ACI 318 yielded better predictions particularly when the bending moment is dominant. The provisions set by EC2-2 and ACI 318 yielded better predictions than the other methods for the entire range of columns analysed. The mean $P_{ECP}/P_t$ and $P_{ACI}/P_t$ ratios were 0.81 and 0.87, respectively, with CoVs of 7.8% and 11.8%, respectively.

Specimens investigated by Lee and Son (2000)

Lee and Son (2000) carried out extensive experimental work on well-confined square column specimens to investigate their structural behaviour under eccentric loading. The main variables included were concrete compressive strength, longitudinal reinforcement ratio, load eccentricity and slenderness ratio.
A high volumetric ratio of lateral reinforcement of 2.04% was used with a yield strength of 340 MPa. The concrete strengths were 34.9, 41.8, 70.4 and 93.2 MPa. The analysis was performed for HSC column specimens. The specimen details and results are summarised in Table 3.

Table 3 shows excellent correlation of the ultimate load results, with a mean predicted-to-experimental ultimate load ratio of 0.95 and a CoV of 10.5%, thus indicating the credibility of the confinement model adopted. The confined concrete strength, $f_{cc}$, for test specimens was shown to range from 1.14 to 1.21 of the unconfined concrete strength, $f_{cu}$. Furthermore, satisfactory predictions of the columns’ deflection at failure were realised.

ECP-203 tended to overestimate the column capacity, particularly for specimens with $li$ = 40. The recorded deflection for these specimens was as high as 53% of the initial eccentricity. It is apparent that overlooking second-order effects in the analysis of HSC short specimens with $li$ ratios close to the limiting value of 50 is inaccurate. The upper limit of the slenderness ratio set by ACI 318 for short braced columns bent in single curvature was 22. This may justify the superior results shown in Table 3 for ACI 318 compared with those of ECP-203. Reliance of the additional load eccentricity considered in the ACI 318 analysis on the initial eccentricity is an added factor. ACI 318 implicitly adopts an almost linear stress-strain relationship for HSC, in agreement with the experimental evidence. The mean value of the ultimate load ratio was 1.11 with a CoV of 11.4% for the ECP-203 predictions compared with 0.89 and 18.8% for the ACI 318 results.

On the other hand, the EC2 slenderness limit for short braced columns bent in single curvature is dependent on the axial load level, the mechanical reinforcement ratio and the creep coefficient. The upper limit of the slenderness ratio for short braced columns is a maximum of 10.78, justifying the conservativeness of the EC2 results shown in Table 3. Specifically, the EC2-2 procedure yielded the least ultimate load variation (the mean $P_{EC2-2}/P_{t}$ ratio was 0.91 with a CoV of 8.7%). The methods based on nominal stiffness indicated higher CoVs (18.8% and 19.8% for ACI 318 and EC2-1, respectively). It should be noted that the mid-height deflection estimated by EC2-2 was mainly dependent on the slenderness ratio, similar to the ECP-203 procedure. It is believed that the overestimation of the ECP-203 method for columns having practical reinforcement ratios up to 2% is due to the use of a stress-strain relationship for normal-strength concrete and to the upper limit of the slenderness ratio for short braced columns ($li$ = 50).

**Conclusions**

The proposed model for the strength analysis of eccentrically loaded pin-ended braced HSC columns assumes that column deflection is dependent on the initial eccentricity, the level of applied load and a buckling load based on variable column stiffness. The model accepts material failure and instability as possible modes of column failure. The model was verified by comparing its results with experimental results of 58 column specimens taken from the literature with different variables (i.e. slenderness ratios, concrete compressive strengths, volumetric ratios of transverse reinforcement and longitudinal reinforcement ratios). The columns were tested under eccentric loading with initial eccentricity ratios ($ei/h$) ranging between 0.09 and 0.54. In addition, the model results were also compared with those of current building codes. The following conclusions can be drawn from the results of this study.

(a) The predicted behaviour (load-carrying capacity and load-deflection characteristics) was in excellent agreement
Table 3. Column specimens tested by Lee and Son (2000) – details and ultimate response

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$f_c'$ (MPa)</th>
<th>$\rho$%</th>
<th>$l_i/l_e$</th>
<th>$e/h$</th>
<th>$P_{uc}$ (kN)</th>
<th>$\Delta_{uc}$ (mm)</th>
<th>$P_{pt}$ (kN)</th>
<th>$P_{pt}/P_{uc}$</th>
<th>$P_{ECP}/P_{pt}$</th>
<th>$P_{EC2-1}/P_{pt}$</th>
<th>$P_{EC2-2}/P_{pt}$</th>
<th>$P_{ACF}/P_{pt}$</th>
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<td>2.2</td>
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<td>529</td>
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</table>

*The yield strength of longitudinal reinforcement was 319–397 MPa
The cube strength of concrete was taken as 1·25

with the test results. The mean predicted-to-experimental ultimate load ratio was 0.94 with a CoV of 10.8%.

(b) Conservative predictions of the ultimate behaviour were obtained using the EC2 (BSI, 2004) procedures. The results following the nominal curvature procedure (EC2-2) were superior compared to those based on the nominal stiffness procedure (EC2-1).

(c) Contrary to the ACI 318 (ACI, 2014) code results, the strength predictions based on the ECP-203 code provisions were generally not conservative. The significance of second-order effects in ECP-203 (HRBC, 2007) is subject to the load eccentricity ratio, $e/h$. Moreover, the upper limit of the slenderness ratio for short braced columns ($l_i/l_e \approx 50$) needs to be lowered.

(d) Neglecting the tension-stiffening effect resulted in an underestimated load-carrying capacity, particularly when the bending moment was dominant.

(e) The mid-height deflection predicted by ECP-203 was found to be independent on the neutral axis position, causing overestimation of the load-carrying capacity for columns with high longitudinal reinforcement content.

Acknowledgement

Dr S. Jones, University of Liverpool, is acknowledged for proofreading the manuscript.

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